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16. Abstract

This report describes research on potential improvements to the software used with the NASA 49.25 MHZ wind profiler located at Kennedy Space Center. In particular, this report provides the analysis and results of a study to (1) identify preferred mathematical techniques for the detection of atmospheric signals that provide wind velocities which are obscured by natural and man-made sources, and (2) to analyze one or more preferred techniques to demonstrate proof of the capability to improve the detection of wind velocities.

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DISCLAIMER

None of the findings, conclusions or recommendations of this study are endorsed by Marshall Space Flight Center.

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1.0 INTRODUCTION AND BACKGROUND

This report is submitted to the NASA, Marshall Space Flight Center's (MSFC's), Space Sciences Laboratory, in conjunction with the deliverable requirements of NASA Purchase Order H-08074D, dated August 30, 1991, entitled "Profiler Signal Detection Improvements." Under the tasking provisions of this Purchase Order, an investigation of atmospheric signal detection algorithms was undertaken for the potential improvement of the predication of wind aloft streamline profiles. Such improved algorithm detection capabilities are needed in NASA's transitioning wind profiling radar technology for detecting atmospheric signals that are occasionally obscured by natural and man-made interference.

The justification for this activity resulted from a NASA review of many profiler spectral samples which showed that there is a weak, but identifiable, atmospheric signal in the spectral power information of the 49.25 Mhz TYCHO Technology Inc. radar currently installed at Kennedy Space Center. This prototype system has an average power-aperture of 10^8 WM^2 and operates with an 8 microsecond pulse consisting of 1 microsecond code elements. This provides a range resolution of 150 meters over 112 range gates. The lowest range gate altitude of this system is located at 2 kilometers and the maximum range gate is located at 18.65 kilometers. Consequently, the range gate altitudes, in meters, are given by the expression $2000+(j-1)*150$, for $j = 1,2,...,112$ where j equals the range gate number.

The phased array antenna of this radar system is composed of coaxial-colinear elements which provide three radar beams or modes. The beam configuration of this system consists of two beams tilted 15^o off the zenith on azimuths of 135^o (mode 1) and 45^o (mode 2), and the third beam (mode 3) is vertical. In operation, a pulse repetition period of $160x10^{-6}$ seconds is utilized and a cycle is completed by integrating each beam for one minute. For modes 1 and 2, the real time radar processor coherently integrates 320 pulses in the time domain; then, a Fast Fourier Transform (FFT) is applied to this time domain data to obtain a 256 point (frequency domain) power spectral estimate. Four sets of these estimates are then averaged to provide a final one minute estimate. In contrast, only one power spectral estimate is formed over the entire minute for mode 3 by averaging 1400 pulses. Consequently, modes 1 and 2 are capable of achieving velocity resolutions of 0.232 m/s over the ±29.6 m/s range, and mode 3 is capable of achieving a velocity resolution of 0.05 m/s over ±6.7 m/s.

Under this current processing scheme, 256 power spectral frequency bins are involved where the zero central frequency (bin 128) corresponds to the transmitted frequency. As a result, the Doppler shifted frequency bins 129-256 correspond to motions toward the profiler, and the frequency bins 1-127 correspond to motions away from the profiler. Furthermore, the TYCHO Technology, Inc. software supplied with the prototype system computes the radial range gate wind velocity component by the so-called "moment method." In this computational method, the largest range gate power spectral peak is integrated over a frequency interval with interval boundaries on either side of the largest peak that have power amplitudes equal to the average range gate power noise level. The centroid frequency component of this area is taken as the Doppler frequency shift which corresponds to the radial velocity estimate. This computational methodology is based on the premise that the power returned in this interval width is a measure of the degree of turbulence in the range gate volume.

In actual practice many range gates have their largest power spectral peak at the central frequency, i.e., frequency bin position 128. For the lower range gates, this effect is often caused by reflections from solid objects (buildings) appearing in the sidelobes of the antenna pattern or local ground clutter effects associated with non-uniform ground heating mechanisms. Consequently, there are many technical issues involved with the measurement of wind aloft profiles from electromagnetic energy scattering. The most apparent issue is that atmospheric signals are occasionally obscured in the presence of natural and man-made interference. For example, interference associated with lightning or aircraft detections within the scattering volume provide frequency intervals of abnormal range gate power amplitudes. In the case of lightning induced interference, the potential of total range gate power spectra contamination is theoretically predictable. Aircraft detections, on the other, tend to locally contaminate the power spectra altitude over time-varying frequency windows. Combining these isolated interference sources with the existing potential of thermal, atmospheric and cosmic noise interference provides the possibility of generating totally uncorrelated range gate power spectra.

2.0 PREREQUISITE ALGORITHM ASSUMPTIONS

These implications dictate that more sophisticated data algorithms must be developed to take full advantage of this technology. To accommodate this situation, a preliminary investigation of prerequisite algorithm requirements was undertaken. During this investigation, the obvious question of algorithm evaluation criteria was raised. This prompted the following enumeration of prerequisite algorithm requirements and considerations:

- Spectral data filtering should be provided by unbiased statistical estimators that account for surrounding range gate power fluctuations.
- The precision of range gate spectral power measurements may vary because many factors have an indirect influence on these measurements.
- Background noise data irregularities cannot be normalized.
- Data filtering must be conducted to reduce or eliminate the magnitude of probable data error uncertainties.

3.0 WEIGHTED MEAN OF MINIMUM VARIANCE ESTIMATOR

By definition, a statistical estimator, $\hat{u}$, is unbiased, if $E(\hat{u}) = \theta$. This property merely states that the random variable $\hat{u}$ possesses a distribution whose mean is the parameter θ being estimated. To place this concept in the context of the current problem, suppose that $X_{i,j}$ denotes the power amplitude of the j^{th} range gate and i^{th} frequency, for a selected beam. Here, $i = 1,2,\ldots,256$ and $j = 1,2,\ldots,112$. Now restrict j to a set of $2n+1$ range gate amplitudes, by taking n range gates on either side of the k^{th} range gate of interest. By letting $j = k-n, k-n-1, \ldots, k, \ldots, k+n-1, k+n$ one obtains $2n+1$ power amplitude range gate observations for the i^{th} range gate frequency. If each of these observations are assumed to have the same expectation, one can assume that subsequent range gate velocities are linearly correlated. This condition is imposed by considering the following linear combination of $2n+1$ random variables:

$$\hat{u}_i = \sum_{j=k-n}^{k+n} a_j X_{i,j}.$$

Since $E(\hat{u}_i)$ must equal θ for $\hat{u}_i$ to be unbiased, it is customary to impose a measure of the variability of $\hat{u}_i$ about θ. This can be accommodated by forcing the condition of minimum variance. To introduce this property of unbiasedness and minimum variance, let

$$E(\hat{u}_i) = E\left(\sum_{j=k-n}^{k+n} a_j X_{i,j}\right) = \sum_{j=k-n}^{k+n} a_j E(X_{i,j}) = \theta \sum_{j=k-n}^{k+n} a_j.$$

This formulation introduces the constraint that

$$\sum_{j=k-n}^{k+n} a_j = 1$$

since $E(\hat{u}_i)$ must equal θ for $\hat{u}_i$ to be unbiased. Furthermore, the variance of $\hat{u}_i$ is given by the following expression:

$$V(\hat{u}_i) = V\left(\sum_{j=k-n}^{k+n} a_j X_{i,j}\right) = \sum_{j=k-n}^{k+n} a_j^2 V(X_{i,j})$$

$$= \sum_{j=k-n}^{k+n} a_j^2 \sigma_j^2.$$

To determine the values of the a_j coefficient weights which make the variance of $\hat{u}_i$ a minimum, one may write the sum of these weights in the following form:

$$a_{k+n} + \sum_{j=k-n}^{k+n-1} a_j = \sum_{j=k-n}^{k+n} a_j = 1;$$

so that

$$a_{k+n} = 1 - \sum_{j=k-n}^{k+n-1} a_j .$$

Thus, the variance of $\hat{u}_i$ can be written in terms of the first 2n+1 weights by the expression,

$$\begin{aligned} V(\hat{u}_i) &= \sum_{j=k-n}^{k+n-1} a_j^2 \sigma_j^2 + a_{k+n}^2 \sigma_{k+n}^2 \\ &= \sum_{j=k-n}^{k+n-1} a_j^2 \sigma_j^2 + [1 - \sum_{j=k-n}^{k+n-1} a_j]^2 \sigma_{k+n}^2 \\ &= \sum_{j=k-n}^{k+n-1} a_j^2 \sigma_j^2 + [\ 1 - 2 \sum_{j=k-n}^{k+n-1} a_j + (\sum_{j=k-n}^{k+n-1} a_j)^2\ ] \sigma_{k+n}^2 . \end{aligned}$$

To find the values of a_j which make this variance a minimum, one must differentiate with respect to a_j and equate to zero:

$$\begin{aligned} \frac{dV(\hat{u}_i)}{da_j} &= 2\ [\ a_j \sigma_j^2 - (\ 1 - \sum_{j=k-n}^{k+n-1} a_j\ ) \sigma_{k+n}^2\ ] \\ &= 2a_j \sigma_j^2 - 2a_{k+n} \sigma_{k+n}^2 \\ &= 0. \end{aligned}$$

Upon solving this last equation for a_j and letting j take on the values k-n,k-n+1,...,k,...,k+n-1 one obtains the expression

$$a_j = \frac{a_{k+n} \sigma_{k+n}^2}{\sigma_j^2} .$$

However, this relationship is obviously true for j = k+n also. Consequently, one can sum over j = k-n,k-n-1,...,k+n to obtain

$$\sum_{j=k-n}^{k+n} a_j = a_{k+n} \sigma_{k+n}^2 \sum_{j=k-n}^{k+n} \frac{1}{\sigma_j^2} .$$

Since the sum of these a_j coefficient weights equals 1,

$$a_j = \frac{1}{\sigma_j^2 \sum_{j=k-n}^{k+n} W_j}$$

for $j = k-n, k-n+1, \ldots, k, \ldots, k+n-1, k+n$, where $W_j = 1/\sigma_j^2$. Upon substituting these coefficient weights into the above variance equation,

$$V(\hat{u}_i) = \frac{1}{\sum_{j=k-n}^{k+n} W_j}$$

equals the minimum variance. Here one must realize that σ_j^2 equals the j^{th} range gate power amplitude sample variance,

$$\frac{\sum_{i=1}^{m} (X_{i,j} - X_m)^2}{m-1} \,.$$

where $m = 256$ and X_m equals the arithmetic mean of the $X_{i,j}$ power amplitude observations, i.e.,

$$X_m = \frac{1}{m} \sum_{i=1}^{m} X_{i,j}.$$

In summary, this unbiased weighted mean of minimum variance formulation provides the capability to statistically construct a mean range gate power spectrum which accounts for adjacent range gate data fluctuations. The compromise associated with this approach resides in the assumption that the wind velocity is assumed to be linearly correlated over subsequent range gate volumes. However, the main disadvantage associated with this formulation is that the magnitude of persistent non-canceling range gate power amplitude fluctuations and irregularities are not systematically removed or filtered out. That is, this formulation does not reduce or eliminate non-canceling range gate probable data error uncertainties. Consequently, additional data filtering or smoothing is required.

4.0 DATA FILTER SELECTION

To help understand the fundamental data filtering requirement, a literature search was conducted of approximately 43 different sources, which revealed two general points of interest:

- o Moving average data filters contain empirical elements that do not appear to be logically necessary.
- o The problem of data filtering belongs to the mathematical theory of probability.

4.1 MOVING AVERAGE FILTERS

The utilization of moving average data filters, such as simple moving average or weighted exponential moving average, contain empirical elements that do not appear to be logically necessary. Specifically, each of these filters are based on the assumption that the true signal can be expressed over a finite time window by a preselected functional curve type. For instance, the simple n point moving average filter gives the average of the n most recent observations and is equivalent to fitting a first degree polynomial (line) to the observations evaluated at the observation window midpoint. If the process is constant and n is large, the simple moving average estimate will be stable. However, if the process is changing, a small value of n is needed for rapid response. Furthermore, each of these moving average filters have a graduated weighting scheme over their finite data smoothing window. Apart from mere convenience, these empirical elements are theoretically questionable.

4.2 PROBABILISTIC FILTER IMPLICATIONS

It is interesting to note that the arbitrary selection or need for these empirical elements can be removed if one views the filtering process in terms of mathematical probability theory. One such probability approach is to assume that the data observation irregularities can be viewed in terms of the hypothesis that data observation uncertainties are distributed in accordance with the normal probability law of errors. In theory, this statistical graduation methodology has the capability to utilize all of the available observations to obtain the "most probable" set of observations, in comparison to the selection of an arbitrary data window smoothing domain. In situations where only erroneous isolated peaks are present in the data, this approach should provide extremely reliable signal reconstruction estimates. This is a direct result of the fact that these isolated peaks are a low probability event. By comparison, the situation of having random errors in the data observations (i.e.,errors of zero mean with serially uncorrelated root-mean-square values) should also provide for extremely reliable signal reconstruction. This performance prediction is a direct result of the inductive antecedent probability that if the observations had been more accurate, the unfiltered data observations would have been smooth.

5.0 NORMAL PROBABILITY LAW FILTER DEVELOPMENT

Under this proposed filtering approach, the basic provisions of the Fundamental Theorem of Inductive Probability apply. Unfortunately, this theorem imposes the embedded assumption that the measurement precision is the same for all data observations. To understand the physical implications of this assumption, suppose one is concerned with n equidistant data observations, u_n, that are affected with uncertainties or irregularities due to accidental errors of observation. In other words, when the u_n data is plotted as a function of n, the points do not lie on a smooth curve, although there is a strong antecedent probability that if the observations had been more accurate the curve would be smooth. Here, the term "smooth" may be given a more precise definition by interpreting it to mean that the third order forward data differences, $\delta^3 u_j = u_j - 3u_{j-1} + 3u_{j-2} - u_{j-3}$, must be small.

To mathematically express this concept of smoothness, the notation of forward differences is used. This notation is based on the operator δ, which expresses the differences of equally spaced data observations. In this notation, the coordinates of a point are given by (t_k, u_k) where the t_k abscissa are equally spaced values with h delta spacing (i.e., $t_{k+1} = t_1 + (k-1)h$) and the ordinates, u_k, are expressed in the following functional notation, $u(t_k) = u_k$. In this notation, the first forward difference of a tabular point $u(t_k)$ is defined as

$$\delta u(t_k) = u(t_k + h) - u(t_k) \quad \text{or} \quad \delta_k^{\,1} = u_{k+1} - u_k .$$

As a result, forward differences of various orders are given by the following general expressions:

$$\delta_k^{\,1} = u_{k+1} - u_k,$$

$$\delta_k^{\,2} = u_{k+2} - 2u_{k+1} + u_k,$$

$$\delta_k^{\,3} = u_{k+3} - 3u_{k+2} + 3u_{k+1} - u_k,$$

$$\delta_k^{\,4} = u_{k+4} - 4u_{k+3} + 6u_{k+2} - 4u_{k+1} + u_k,$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\delta_k^{\,n} = \sum_{j=0}^{n} (-1)^j \left[\frac{n!}{j!\,(n-j)!}\right] u_{k+n-j} .$$

Now consider the hypothesis:

> The true value, which should have been obtained by the observation u_1, lies between $u_1^{\,1}$ and $u_1^{\,1}+\blacktriangle$, where $\blacktriangle$ is a small constant (e.g., one unit in the last decimal place used in the data observation measurements), so that the true value which should have been obtained by

the measurement for u_2, lies between u_2^1 and $u_2^1+\blacktriangle$, and so on, up to u_n; which lies between u_n^1 and $u_n^1+\blacktriangle$.

This shall be called "Hypothesis H." However, before the observations have been made, one has no probability knowledge of the situation, other than the above hypothesis and the definition of smoothness. Nevertheless it is possible to formulate the degree of smoothness of the sequence of u_i measurements, by taking the sum of squares of the third differences

$$S = (u_4^1-3u_3^1+3u_2^1-u_1^1)^2 + (u_5^1-3u_4^1+3u_3^1-u_2^1)^2 + \dots + (u_n^1-3u_{n-1}^1+3u_{n-2}^1-u_{n-3}^1)^2$$

which will be large for sequences of rough data. Therefore, S may be called the measure of roughness of the sequence. In theory, this measure can be extended to the case where the observations are not taken at equidistant values of time by taking, instead of S, the sum of the squares of the third divided differences of the graduated values.

By analogy to the normal probability law of errors, one may suppose that the a priori probability of Hypothesis H is

$$C\exp(-B^{2S})\blacktriangle^n \quad \dots\dots\dots\dots\dots\dots\dots (5.0\text{-}1)$$

where C and B are constants. Next consider the a priori probability that the observations will be $u_1,u_2,u_3,\dots,u_n$, under the assumption that Hypothesis H is true.

Since the true value of the first observed quantity is u_1^1 (under this hypothesis), the probability that a value between u_1 and $u_1+\blacktriangle$ will actually be observed is

$$h_1/\sqrt{\pi}\ \exp(-h_1^{2\beta})\blacktriangle,$$

under the postulated normal probability law of errors, where h_1 is a constant which measures the precision with which this observation can be made and $\beta = (u_1-u_1^1)^2$. Similarly, the probability that a value between u_2 and $u_2+\blacktriangle$ will actually be obtained for the second observed value is

$$h_2/\sqrt{\pi}\ \exp(-h_2^{2\alpha})\blacktriangle$$

where $\alpha = (u_2-u_2^1)^2$ and h_2 is the measure of precision of this second observation. Thus, under the assumption that Hypothesis H is true, the a priori probability that the first observed quantity will lie between u_1 and $u_1+\blacktriangle$, the second observed quantity between u_2 and $u_2+\blacktriangle$, and so on, is

$$[h_1h_2h_3\dots h_n/(\sqrt{\pi})^n]\ \exp(-\Omega)\blacktriangle^n \quad \dots\dots\dots\dots\dots (5.0\text{-}2)$$

where $\quad \Omega = h_1^2(u_1-u_1^1)^2 + h_2^2(u_2-u_2^1)^2 + \dots + h_n^2(u_n-u_n^1)^2.$

The sums S and Ω enable one to numerically express the smoothness of the graduated values, as well as the fidelity of the graduated to ungraduated values.

To accommodate this formulation, the Fundamental Theorem of Inductive Probability must be utilized. That is, suppose that a certain observed phenomenon may be accounted for by any one of a certain number of hypotheses. Then suppose that the probability of the s^{th} hypothesis (based on a priori information before the phenomenon is observed) is p_s. Under this situation, the probability of the observed phenomenon, based on the assumed truth of the s^{th} hypothesis, is P_s. Consequently, the probability of the s^{th} hypothesis is

$$p_s P_s / \Sigma(p_{sj} P_{sj})$$

where Σ denotes the summation over all the hypotheses $j = 1,2,3,\ldots$. It follows from this expression that before the phenomenon was observed the most probable hypothesis was that for which p_s was the greatest. However, the most probable hypothesis after the phenomenon has been observed is that for which the product $p_s P_s$ is the greatest. Applying this condition to the case under consideration, equations (5.0-1) and (5.0-2) may be combined to obtain

$$[C h_1 h_2 h_3 \ldots h_n / (\surd\pi)^n] \exp(-B^{2S-\Omega}) \blacktriangle^{2n}.$$

Since this expression is a maximum when $B^{2S-\Omega}$ is a minimum, the most probable set of values, $u_1^1, u_2^1, u_3^1, \ldots, u_n^1$, are obtained when $B^{2S-\Omega}$ is a minimum.

5.1 ANALYTICAL MINIMUM FORMULATION

Writing down the ordinary difference equation conditions for a minimum, one obtains the equations:

$$h_1^2 u_1 = h_1^2 u_1^1 - B^2\delta^3 u_1^1$$

$$h_2^2 u_2 = h_2^2 u_2^1 + 3B^2\delta^3 u_2^1 - B^2\delta^3 u_2^1$$

$$h_3^2 u_3 = h_3^2 u_3^1 - 3B^2\delta^3 u_1^1 + 3B^2\delta^3 u_2^1 - B^2\delta^3 u_3^1$$

$$h_4^2 u_4 = h_4^2 u_4^1 + B^2\delta^3 u_1^1 - 3B^2\delta^3 u_2^1 + 3B^2\delta^3 u_3^1 - B^2\delta^3 u_4^1$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$h_n^2 u_n = h_n^2 u_n^1 + B^2\delta^3 u_{n-3}^1.$$

By making the simplifying assumption that the measure of precision is the same for all data observations, i.e.($h_1 = h_2 = h_3 = \ldots = h_n$), one imposes a constant measure of precision across all observations. If this is not the case, one can graduate some function of u, such as ln(u), instead of u, choosing this function so that its measure of precision has nearly the same value for all values of the argument. However, under the equal precision assumption, one may substitute $h_j^2 = \epsilon B^2$ for $j = 1,2,3,\ldots,n$ into each of the above equations.

Here, ϵ denotes the index of precision generally associated with the normal probability density function,

$$f(t) = \frac{\epsilon}{\sqrt{\pi}} \exp(-\epsilon^2 t^2).$$

(See Section 5.5, NORMAL PROBABILITY INDEX OF PRECISION FORMULATION, for a detailed discussion of this function.)

Under the above noted substitution the equations have the form,

$$\begin{aligned}
\epsilon u_1 &= \epsilon u_1{}^1 - \delta^3 u_1{}^1 \\
\epsilon u_2 &= \epsilon u_2{}^1 + 3\delta^3 u_1{}^1 - \delta^3 u_2{}^1 \\
\epsilon u_3 &= \epsilon u_3{}^1 - 3\delta^3 u_1{}^1 + 3\delta^3 u_2{}^1 - \delta^3 u_3{}^1 \\
\epsilon u_4 &= \epsilon u_4{}^1 + \delta^3 u_1{}^1 - 3\delta^3 u_2{}^1 + 3\delta^3 u_3{}^1 - \delta^3 u_4{}^1 \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\epsilon u_n &= \epsilon u_n{}^1 + \delta^3 u_{n-3}{}^1.
\end{aligned} \qquad \ldots(5.1\text{-}1)$$

Now all of these equations, except the first three and last three, are of the form

$$\epsilon u_j = \epsilon u_j + \delta^6 u_{j-3}{}^1.$$

This can be directly verified analytically from the forward difference equations provided earlier. If one introduces a quantity $u_0{}^1$, such that $\delta^3 u_0{}^1 = 0$, the third equation,

$$\epsilon u_3 = \epsilon u_3{}^1 - \delta^6 u_0{}^1,$$

is of this same form. Similarly, the first two and last three equations can be brought to this form, by introducing the new quantities $u_{-1}{}^1, u_{-2}{}^1, u_{n+1}{}^1, u_{n+2}{}^1$, and $u_{n+3}{}^1$; such that $\delta^3 u_{-1}{}^1 = 0$, $\delta^3 u_{-2}{}^1 = 0$, $\delta^3 u_{n-2}{}^1 = 0$, $\delta^3 u_{n-1}{}^1 = 0$, and $\delta^3 u_n{}^1 = 0$. Thus the graduated values $u_j{}^1$ satisfy the linear difference equation

$$\epsilon u_j{}^1 - \delta^6 u_{j-3}{}^1 = \epsilon u_j, \quad \ldots\ldots\ldots\ldots(5.1\text{-}2)$$

being in fact the particular solution of this equation which satisfies the six terminal conditions

$\delta^3 u_{-2}{}^1 = 0$, $\delta^3 u_{-1}{}^1 = 0$, $\delta^3 u_{-2}{}^1 = 0$, $\delta^3{}_{n-2} = 0$, $\delta^3 u_{n-1}{}^1 = 0$, and

$\delta^3 u_n{}^1 = 0$,

which provides the following relationships

$\delta^4 u_{-2}{}^1 = 0$, $\delta^4 u_{-1}{}^1 = 0$, $\delta^5 u_{-2}{}^1 = 0$, $\delta^4 u_{n-2}{}^1 = 0$, $\delta^4 u_{n-1}{}^1 = 0$, and

$\delta^5 u_{n-2}{}^1 = 0$. ....(5.1-3)

5.2 CONSERVATION THEOREM OF MOMENTS

By the summation of equation (5.1-2)

$$\begin{aligned}\epsilon(u_1{}^1+u_2{}^1+\ldots+u_n{}^1)-\epsilon(u_1+u_2+\ldots+u_n) &= \delta^6 u_{-2}{}^1+\delta^6 u_{-1}{}^1+\ldots+\delta^6 u_{n-3}{}^1\\ &= \delta^5 u_{n-2}{}^1-\delta^5 u_{-2}{}^1\\ &= 0,\end{aligned}$$

as a result of the 3rd and 6th particular solution values of equation (5.1-3).

Therefore,

$$u_1{}^1+u_2{}^1+\ldots+u_n{}^1 = u_1+u_2+\ldots+u_n \ .$$

Moreover, by equation (5.1-2)

$$\epsilon\sum_{j=1}^{n} j u_j{}^1 - \epsilon\sum_{j=1}^{n} j u_j = \sum_{k=-2}^{n-3}(k+3)\delta^6 u_k{}^1 = 0$$

$$\text{or } u_1{}^1+2u_2{}^1+\ldots+nu_n{}^1 = u_1+2u_2+\ldots+nu_n.$$

At this point it should be noted that equation (5.1-2) can be written in the form

$$\epsilon j^2(u_j{}^1-u_j) = j^2\delta^6 u_{j-3}{}^1$$

for $j = 1,2,3,\ldots,n$. Upon expanding over j and adding, these difference equations,

$$\begin{aligned}\epsilon\sum_{j=1}^{n} j^2 u_j{}^1 - \epsilon\sum_{j=1}^{n} j^2 u_j &= \sum_{k=-2}^{n-3}(k+3)^2\delta^6 u_k{}^1\\ &= \delta^6 u_{-2}{}^1+2^2\delta^6 u_{-1}{}^1+\ldots+n^2\delta^6 u_{n-3}{}^1\end{aligned}$$

$$= n^2\delta^5 u_{n-2}{}^1-(2n-1)\delta^4 u_{n-2}{}^1+2\delta^3 u_{n-2}{}^1-\delta^3 u_{-2}{}^1-$$
$$\delta^3 u_{-1}{}^1$$
$$= 0.$$

Therefore, $u_1{}^1+2^2u_2{}^1+\ldots+n^2u_n{}^1 = u_1+2^2u_2+\ldots+n^2u_n$, which shows that the moments of order 0,1,and 2 are the same for the filtered and unfiltered data. In other words, the filtered and unfiltered graphs have equal area, center of gravity, and moments of inertia about any line parallel to the observation ordinates.

5.3 GRADUATED DIFFERENCE EQUATION SOLUTION

In this section, the solution of the difference equation

$$\epsilon u_j{}^1 - \delta^6 u_{j-3}{}^1 = \epsilon u_j,$$

subject to the six terminal conditions outlined in Section 5.1 will be investigated. To start this discussion, assume that the normal probability index of precision, ϵ, is given, so that a general solution can be obtained in terms of symbolic operator notation. Here, it is assumed that ϵ is a positive non-zero constant. Since δ = E-1 in operator notation, this difference equation can be written as:

$$[\{(E-1)^6-\epsilon E^3\}/E^3]u_j{}^1 = -\epsilon u_j$$

or

$$u_j{}^1 = -\epsilon[E^3/\{(E-1)^6-\epsilon E^3\}]u_j. \quad \ldots\ldots\ldots(5.3\text{-}1)$$

In this form, considerations of symmetry lead one to expect that each u^1 will be given as a linear function of the u's, with coefficients symmetrically placed about the central term. This suggests expanding the right-hand side in powers of both E and E^{-1}. Such an expansion is readily seen to be possible, because the complex equation $(z-1)^6-\epsilon z^3 = 0$ has its six roots in reciprocal pairs, so that three are less in modulus than unity. Hence, the equation $[(z-1)^6-\epsilon z^3]^{-1}$ may be expanded as a Laurent series, convergent for z = 1, and the coefficients may be found by the usual theory. The fact that these coefficients are symmetrical follows from the fact that the operator is left unaltered by the substitution of E^{-1} for E.

Utilization of the theory of functions of a complex variable may be avoided by having recourse to the theory of partial fractions. That is, if the operator in question is resolved into six partial operators by this theory, three of these operators may be expanded in powers of E and the other three in powers of E^{-1}. As a result, the addition of these operators provide the complete expansion.

Either of these methods leads to the same graduation formula, namely:

$$u_j{}^1 = k_0u_j + k_1(u_{j+1}+u_{j-1}) + k_2(u_{j+2}+u_{j-2}) + \ldots, \quad \ldots.(5.3\text{-}2)$$

where

$$k_n = -\epsilon\Sigma \frac{(s_1)^{n+2}}{(s_1-s_2)(s_1-s_3)(s_1-(s_1)^{-1})(s_1-(s_2)^{-1})(s_1-(s_3)^{-1})}.$$

Here s_1, s_2, s_3 are the three roots less in modulus than unity of the equation $(z-1)^6-\epsilon z^3 = 0$, and Σ denotes interchange of s_1, s_2, s_3, followed by summation. One of the roots is evidently real; hence, these roots may be written as r_1, $r_2\exp(i\theta)$, and $r_2\exp(-i\theta)$, so that the above equation becomes:

$$k_j = -\frac{1}{r_1{}^2-2r_1r_2\mathrm{Cos}[\theta]+r_2{}^2} \cdot \delta_j{}^3 \cdot \{r_1{}^{j-1}+\frac{Q_j}{\mathrm{Sin}(\theta)}\}$$

where

$$Q_j = r_2{}^{j-2} \cdot [r_2\mathrm{Sin}((j-2)\theta)-r_1\mathrm{Sin}((j-1)\theta)]$$

for $j = 1,2,3,\ldots,n$.

This expression provides the necessary provisions needed to compute the k_j coefficients needed in the graduation formula of equation (5.3-2) for representative values of the normal probability index of precision.

5.4 EXTENDED AUXILIARY DATA POINT PROVISIONS

An inherent defect is present in this formulation unless some means can be developed to graduate the complete set of data observations. To investigate this possibility, consider attaching auxiliary data points to each end of the observation data set $u_1, u_2, u_3, \ldots, u_n$. The existence of this possibility is implied by the graduation formula of equation (5.3-2). To accommodate this addition of auxiliary data points beyond the domain of the original data observations, the requirement can be imposed that the third differences involving these extended points must be zero. Insight into this formulation can be obtained by first introducing the auxiliary points, $u_{n+1}, u_{n+2}, u_{n+3}, \ldots$ and $u_0, u_{-1}, u_{-3}, \ldots$. By differencing the added third differences three times, one obtains

$$\delta^6 u_{n-2}{}^1 = 0,\ \delta^6 u_{n-1}{}^1 = 0,\ \ldots\ ,\ \text{and}\ \delta^6 u_{-3}{}^1 = 0,\ \delta^6 u_{-4}{}^1 = 0\ \ldots\ .$$

Referring to the difference equation (5.1-2), and imposing the condition that $u_i{}^1 = u_i$ and $u_k{}^1 = u_k$ for $i = 0,-1,-2,\ldots$ and $k = n+1, n+2, n+3, \ldots$ forces the condition of having extended graduated and ungraduated values coincide. It also follows that $\delta^3 u_{n+1} = 0$, $\delta^3 u_{n+2} = 0$, ..., and $\delta^3 u_{-3} = 0$, $\delta^3 u_{-4} = 0$, ... , as a result of this condition. Consequently, equation (5.1-2) along with its six terminal conditions combined with the above auxiliary conditions of having the indicated third differences equal to zero, suffices to determine the extended auxiliary data requirements.

If the original number of data observations is not too small (i.e., if it exceeds twenty-five), the extended data may be found as accurately as required by the following method. From the condition that the graduated and ungraduated auxiliary data coincide together with equation (5.3-2) provides

$$u_j = -\frac{\epsilon E^3}{(E-1)^6-\epsilon E^3} u_j$$

or $$\frac{(E-1)^6}{(E-1)^6-\epsilon E^3} u_j = 0, \text{ for } j = n+1,\ n+2,\ \ldots\ .$$

This last equation may be written as,

$$\frac{(E-1)^6}{(E-s_1)(E-s_2)(E-s_3)} \cdot \frac{(E-1)^3}{(E-(s_1)^{-1})(E-(s_2)^{-1})(E-(s_3)^{-1})} u_j = 0. \quad (5.4\text{-}1)$$

To show that u_{n+1}, u_{n+2}, ... are given by the equation formed by retaining only the first operating factor on the left side of the above equation, namely;

$$\frac{(E-1)^3}{(E-s_1)(E-s_2)(E-s_3)} u_j = 0 \ \ldots\ldots\ldots (5.4\text{-}2)$$

one should note that this equation gives u_j uniquely in terms of u_{j-1}, u_{j-2}, ... , since the operator can be expanded in descending powers of E, in the form

$$1 - q_1E^{-1} - q_2E^{-2} - q_3E^{-3} - \ldots ,$$

so that

$$u_j = q_1u_{j-1} + q_2u_{j-2} + q_3u_{j-3} + \ldots ,$$

for $j = n+1, n+2, n+3, \ldots$. From this equation u_{n+1}, u_{n+2}, ... are determined in succession, provided that the terms containing the undetermined extended data points u_0, u_{-1}, u_{-2}, ... are negligible. These values of u_j will satisfy equation (5.4-1) for the operator

$$\frac{(E-1)^3}{(E-(s_1)^{-1})(E-(s_2)^{-1})(E-(s_3)^{-1})}$$

which is expandable in ascending powers of E. Now equation (5.4-1) can be deduced from equation (5.4-2), which holds for $j = n+1$, $n+2$, $n+3$, ...

Lastly, the conditions $\delta^3u_{n+1} = 0$, $\delta^3u_{n+2} = 0$, ... , and $\delta^3u_{-3} = 0$, $\delta^3u_{-4} = 0$, ... will be satisfied since equation (5.4-2) can be written as

$$\frac{(E-1)^3}{(E-s_1)(E-s_2)(E-s_3)} \cdot (E-s_1)(E-s_2)(E-s_3)u_j = 0.$$

Hence $(E-1)^3 u_j = \delta^3 u_j = 0$, for $j = n+1, n+2, n+3, \ldots$. In addition, the values of q_j for $j = 1,2,3,\ldots$ are found to be

$$q_j = -\frac{1}{D}\,\delta_j{}^3 v_j$$

$$= -\frac{1}{D}\,[v_{j+3} - 3v_{j+2} + 3v_{j+1} - v_j]$$

where $D = r_1{}^2 - 2r_1r_2\mathrm{Cos}[\theta] + r_2{}^2$ and

$$v_j = r_1{}^{j-1} + \frac{r_1{}^{j-2}\,\{r_2\mathrm{Sin}((j-2)\theta) + r_1\mathrm{Sin}((j-1)\theta)\}}{\mathrm{Sin}(\theta)}$$

where r_1, $r_2\exp(i\theta)$ and $r_2\exp(-i\theta)$ are the real and complex roots of the equation

$$(z-1)^6 - \epsilon z^3 = 0.$$

To compute these roots, expand $(z-1)^6$ using the r^{th} term expansion formula of the Binomial Theorem,

$$\frac{n!}{(r-1)!(n-r+1)!}\, a^{n-r+1}b^{r-1} \text{ of } (a+b)^n$$

where $a = z$, $b = -1$ an $n = 6$ for $r = 1,2,3,4,5,6,7$. That is,

$$r = 1 \text{ -------- } \frac{6!}{0!6!} z^6 \ (-1)^0 = z^6$$

$$r = 2 \text{ -------- } \frac{6!}{1!5!} z^5 \ (-1)^1 = -6z^5$$

$$r = 3 \text{ -------- } \frac{6!}{2!4!} z^4 \ (-1)^2 = 15z^4$$

$$r = 4 \text{ --------- } \frac{6!}{3!3!} z^3 \; (-1)^3 = -20z^3$$

$$r = 5 \text{ --------- } \frac{6!}{4!2!} z^2 \; (-1)^4 = 25z^2$$

$$r = 6 \text{ --------- } \frac{6!}{5!1!} z^1 \; (-1)^5 = -6z$$

$$r = 7 \text{ --------- } \frac{6!}{6!0!} z^0 \; (-1)^6 = 1.$$

As a result,

$$(z-1)^6 - \epsilon z^3 = z^6 - 6z^5 + 15z^4 - (\epsilon+20)z^3 + 15z^2 - 6z + 1 = 0$$

where r_1 equals the smallest real root of this equation and r_2 equals the modulus (i.e., $r_2 = \sqrt{(c_1^2+c_2^2)}$) of the complex conjugate root pair, $z = c_1 \pm ic_2$, less than unity and

$$\theta = \mathrm{Tan}^{-1}(c_2/c_1).$$

Since the successive values of q_j decrease quite rapidly the number of auxiliary data points needed in practice will depend upon the degree of accuracy required and the rapidity with which the successive graduating coefficients diminish. In theory, only three auxiliary data points at each end need be calculated. The rest can be found by virtue of the constraints that $\delta^3 u_j = 0$ and $\delta^3 u_k = 0$ for $j = n+1, n+2, n+3, \ldots$ and $k = -3, -4, -5, \ldots$, by forming a difference table with zero third differences. Notice, exactly the same process may be applied to both ends. However, the far end may be handled by reversing the order of the original data observations.

5.5 NORMAL PROBABILITY INDEX OF PRECISION FORMULATION

As noted earlier, representative values of the normal probability index of precision, ϵ, may be used to compute the graduated signal reconstruction coefficients, k_j, as well as, the auxiliary data point coefficients, q_k. The underlying consideration is the determination of which values of ϵ are likely to be of practical application. To consider this dilemma, it is necessary to find a means of assigning, in advance, an appropriate value of ϵ. Statistically, this value should be based on the probable error of the sampled data observations, because many unknown factors have an indirect influence on these values. In addition, such a statistical approach has the advantage of preserving the probabilism of this formulation.

Under this approach, one may utilize the analytical form of the normal probability index of precision integral, discussed by Worthing and Geffner*:

$$\int_{-Z}^{Z} f(t)dt = \int_{0}^{Z} f(t)dt$$

1) The total number of deviations, $t = u - \tilde{n}_u$, over the entire range of possible values of t must be equal to the total number of observations, n, i.e.,

$$n = n \int_{-\infty}^{\infty} f(t)dt \quad \text{or} \quad \int_{\infty}^{\infty} f(t)dt = 1.$$

where $\tilde{n}_u$ equals the average value of the original u observations

$$\tilde{n}_u = \frac{1}{n} \sum_{j=1}^{n} u_j.$$

2) The most probable value of $t = 0$, i.e., the value $u = \tilde{n}_u$.

3) When $u = \tilde{n}_u$ or $t = 0$, $f(t)$ is a maximum.

To determine the analytical form of $f(t)$, assume that the values of t can be divided into Δt intervals so that only one value of t occurs in each interval:

$$f(t_1)\Delta t = \text{probability of the deviation } t_1;$$

$$f(t_2)\Delta t = \text{probability of the deviation } t_2;$$

$$\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad \cdot$$

$$f(t_n)\Delta t = \text{probability of the deviation } t_n.$$

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* Worthing, A.G. and Geffner, J.; "Treatment of Experimental Data," John Wiley and Sons, Inc., New York (1943), p.155. This integral is also listed as No. 492 in Pierce, B.O.; "A Short Table of Integrals," Ginn and Co. (1929).

Therefore, the probability of this combination of deviations is given by the expression,

$$W = (\blacktriangle t)^n \underset{j=1}{\overset{n}{E}} f(t_j).$$

Upon taking the logarithm of both sides of this equation:

$$\ln(W) = \Sigma \ln(f(t_j)) + n\ln(\blacktriangle t). \quad ....(5.5\text{-}1)$$

From restriction 2 and the definition of $\tilde{n}_u$ it follows that

$$\sum_{j=1}^{n} t_j = \sum_{j=1}^{n} (u_j - \tilde{n}_u) = \sum_{j=1}^{n} u_j - n\tilde{n}_u = 0.$$

Given this result together with restriction 3), it follows that

$$\frac{d\ln(W)}{d\tilde{n}_u} = 0$$

so that the probability W depends indirectly on $\tilde{n}_u$ through the t_j's. By utilizing indirect differentiation (i.e., the Chain Rule) on equation (5.5-1):

$$\frac{d(\ln W)}{d\tilde{n}_u} = \frac{d(\ln W)}{dt_1} \cdot \frac{dt_1}{d\tilde{n}_u} + \frac{d(\ln W)}{dt_2} \cdot \frac{dt_2}{d\tilde{n}_u} + \ldots + \frac{d(\ln W)}{dt_n} \cdot \frac{dt_n}{d\tilde{n}_u} = 0.$$

Since the last term of equation (5.5-1) is a constant, its derivative is zero, so that each term of this equation involves only one t. This implies that,

$$\frac{d(\ln W)}{d\tilde{n}_u} = \sum_{j=1}^{n} g(t_j) = 0 \quad ....(5.5\text{-}2)$$

where a typical term is defined by the expression

$$g(t_j) = \frac{\dfrac{df(t_j)}{dt_j}}{f(t_j)}. \quad ....(5.5\text{-}3)$$

In arriving at this last result, the fact that

$$\frac{dt_j}{d\tilde{n}_u} = \frac{d(u_j - \tilde{n}_u)}{d\tilde{n}_u} = -1$$

for j = 1,2,3,...,n was utilized. In line with the standard techniques of solving differential equations, each of these $g(t_j)$ terms can be expressed by an infinite series:

$$g(t_j) = b_0 + b_1 t_j + b_2 t_j^2 + b_3 t_j^3 + \ldots \quad \ldots.(5.5\text{-}4)$$

These constant coefficients may be determined from equations (5.5-2) and (5.5-4) by summing the respective $g(t_j)$ series for $j = 1,2,3,\ldots,n$ and setting the resulting sum equal to zero in accordance with the constraint of equation (5.5-2). These operations provide,

$$\sum_{j=1}^{n} g(t_j) = nb_0 + b_1 \sum_{j=1}^{n} t_j + b_2 \sum_{j=1}^{n} t_j^2 + b_3 \sum_{j=1}^{n} t_j^3 + \ldots = 0.$$

In addition, the above equation can be satisfied if each of the individual terms are equal to zero. This may be accomplished by having all of the b's except b_1 equal to zero. This condition is imposed because

$$\sum_{j=1}^{n} t_j = 0.$$

By utilizing equations (5.5-3) and (5.5-4), in conjunction with the above b_j coefficient constraint

$$\frac{df(t_j)}{dt_j} = b_1 t_j f(t_j) \quad \text{or} \quad \frac{df(t_j)}{f(t_j)} = b_1 t_j dt_j.$$

Upon integration,

$$\int \frac{df(t)}{f(t)} = \ln(f(t)) + C_1 = b_1 \int t dt = \frac{b_1 t^2}{2} + C_2.$$

and $\ln(f(t)) = \dfrac{b_1 t^2}{2} + (C_2 - C_1)$ or $f(t) = C \exp(b_1 t^2/2)$

where C is the constant of integration. Since $f(t)$ is known to have a maximum at $t = 0$, and is expected to approach zero for very large positive or negative values of t, b_1 must be negative. Thus, it is appropriate to replace $b_1/2$ by

$$\epsilon^2 = -\frac{b_1}{2}$$

which yields $f(t) = C \exp(-\epsilon^2 t^2)$. The constant of integration, C, may be evaluated by substituting $f(t)$ into restriction 1) and performing the indicated integration, i.e.,

$$\int_{-\infty}^{\infty} f(t) dt = 2 \int_{0}^{\infty} f(t) dt = C \int_{-\infty}^{\infty} \exp(-\epsilon^2 t^2) dt = \frac{C\sqrt{\pi}}{\epsilon} = 1.$$

Therefore, the constant of integration is given by

$$C = \frac{\epsilon}{\sqrt{\pi}} \quad \text{and} \quad f(t) = \frac{\epsilon}{\sqrt{\pi}} \exp(-\epsilon^2 t^2)$$

As a result, a large value of ϵ provides a high, narrow curve for f(t), which corresponds to observations with a high degree of precision. Conversely, a small value of ϵ yields a low, flat curve which corresponds to observations of low precision.

5.6 PROBABLE OBSERVATION ERROR FORMULATION

To formulate the probable observation error, one must establish the probability of obtaining an observation with deviation, t, in the range $-P_e$ to P_e. This probability is found by integrating f(t) from $-P_e$ to P_e and setting this integral equal to ½. However f(t) is a symmetric function about t = 0 so that this integral is twice the integral from 0 to P_e. Hence, the probable error, P_e, of an observation is defined as,

$$\int_{-P_e}^{P_e} f(t)dt = 2 \int_0^{P_e} f(t)dt = \frac{2\epsilon}{\sqrt{\pi}} \int_0^{P_e} \exp(-\epsilon^2 t^2)dt = \tfrac{1}{2}$$

In other words, P_e is that magnitude of deviation whose probability of being exceeded is one-half. This consideration gives:

$$P_e \approx \frac{0.476900036}{\epsilon}.$$

The geometrical interpretation of P_e is that the area under f(t) between the limits $-P_e$ and P_e is equal to ½ the total area under f(t).

Another interesting relationship is that $\sigma = 1/\epsilon\sqrt{2}$, where σ is the standard deviation of the original observations. To derive this relationship, rewrite the standard equation for variance in the form

$$\sigma^2 = \frac{\epsilon}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 \exp(-\epsilon^2 t^2)dt$$

$$= \frac{2\epsilon}{\sqrt{\pi}} \int_0^{\infty} t^2 \exp(-\epsilon^2 t^2)dt$$

$$= \frac{1}{\epsilon\sqrt{\pi}} \int_0^\infty t\exp(-\epsilon^2 t^2)(-2\epsilon^2 t)\,dt$$

and integrate by parts using $U = t$, $dV = -2t\epsilon^2\exp(-\epsilon^2 t^2)$ and $V = \exp(-\epsilon^2 t^2)$ to obtain

$$\sigma^2 = \frac{1}{\epsilon\sqrt{\pi}} \int_0^\infty U\,dV$$

$$= \frac{1}{\epsilon\sqrt{\pi}} \{[t\exp(-\epsilon^2 t^2)]\Big|_0^\infty - \int_0^\infty \exp(-\epsilon^2 t^2)\,dt\}.$$

Clearly, the first term on the right vanishes at both limits and the second term is equal to $\sqrt{\pi}/2\epsilon$, so that

$$\sigma^2 = \frac{1}{2\epsilon^2} \quad \text{or} \quad \sigma = \frac{1}{\epsilon\sqrt{2}} \approx \frac{0.707106781}{\epsilon}.$$

5.7 REPRESENTATIVE GRADUATING COEFFICIENTS

By means of the above methods, the graduating coefficients, k_n, and auxiliary data coefficients, q_n have been evaluated for a number of representative values of the normal probability index of precision, ϵ. Tables of these values to four decimal places are given below:

k_n Coefficients

Graduation Formula:

$$u_j^1 = k_0 u_j + k_1(u_{j+1}+u_{j-1}) + k_2(u_{j+2}+u_{j-2}) + \dots .$$

n	ϵ=0.01	ϵ=0.02	ϵ=0.05	ϵ=0.10	ϵ=0.25	ϵ=1.00
0	0.1570	0.1769	0.2076	0.2347	0.2771	0.3601
1	0.1482	0.1644	0.1873	0.2056	0.2297	0.2604
2	0.1254	0.1329	0.1397	0.1412	0.1356	0.1045
3	0.0948	0.0928	0.0842	0.0721	0.0486	0.0023
4	0.0628	0.0536	0.0359	0.0191	-0.0046	-0.0293
5	0.0341	0.0216	0.0027	-0.0110	-0.0236	-0.0213
6	0.0117	-0.0004	-0.0145	-0.0211	-0.0211	-0.0056
7	-0.0035	-0.0125	-0.0191	-0.0186	-0.0108	0.0032
8	-0.0119	-0.0165	-0.0161	-0.0110	-0.0017	0.0044
9	-0.0148	-0.0151	-0.0099	-0.0035	0.0031	0.0023

10	-0.0138	-0.0110	-0.0038	0.0014	0.0039	0.0003
11	-0.0108	-0.0062	0.0005	0.0034	0.0027	-0.0006
12	-0.0070	-0.0021	0.0027	0.0033	0.0010	-0.0005
13	-0.0034	0.0008	0.0032	0.0022	-0.0001	-0.0002
14	-0.0005	0.0024	0.0026	0.0009	-0.0005	0.0000
15	0.0014	0.0028	0.0016	0.0000	-0.0005	0.0001
16	0.0023	0.0025	0.0006	-0.0004	-0.0003	0.0000
17	0.0025	0.0018	-0.0001	-0.0005	-0.0001	...
18	0.0022	0.0010	-0.0004	-0.0004	0.0001	...
19	0.0017	0.0003	-0.0005	-0.0002	0.0001	...
20	0.0010	-0.0001	-0.0004	0.0000	0.0000	...
21	0.0005	-0.0004	-0.0002	0.0000	...	...
22	0.0000	-0.0004	-0.0001	0.0001	...	...
23	-0.0002	-0.0004	0.0000	0.0001	...	...
24	-0.0004	-0.0003	0.0001	...	...	...
25	-0.0004	-0.0002	0.0001	...	...	...
26	-0.0003	0.0000	0.0001	...	...	...
27	-0.0002	0.0000	...	...	...	...
28	-0.0001	0.0001	...	...	...	...
29	-0.0001	0.0001	...	...	...	...
30	0.0000	0.0001	...	...	...	...
31	0.0000	...	...	...	...	...
32	0.0001	...	...	...	...	...
33	0.0001	...	...	...	...	...
34	0.0001	...	...	...	...	...
35	...	...	...	...	...	...

q_n **Coefficients**

Auxiliary Data Formula:

$$u_{n+j} = q_1 u_{n+j-1} + q_2 u_{n+j-2} + q_3 u_{n+j-3} + \ldots, \quad (j>0)$$

n	ϵ=0.01	ϵ=0.02	ϵ=0.05	ϵ=0.10	ϵ=0.25	ϵ=1.00
1	0.9155	1.0237	1.1847	1.3209	1.5204	1.8615
2	0.4491	0.4337	0.3812	0.3082	0.1523	-0.2545
3	0.1321	0.0626	-0.0600	-0.1750	-0.3468	-0.5842
4	-0.0563	-0.1293	-0.2297	-0.2965	-0.3471	-0.2660
5	-0.1442	-0.1917	-0.2309	-0.2294	-0.1700	0.0302
6	-0.1619	-0.1745	-0.1542	-0.1060	-0.0075	0.1257
7	-0.1374	-0.1199	-0.0633	-0.0027	0.0731	0.0899
8	-0.0937	-0.0579	0.0069	0.0531	0.0810	0.0276
9	-0.0474	-0.0065	0.0452	0.0654	0.0523	-0.0087
10	-0.0084	0.0268	0.0548	0.0512	0.0189	-0.0156
11	0.0185	0.0418	0.0460	0.0281	-0.0031	-0.0088
12	0.0330	0.0425	0.0293	0.0078	-0.0112	-0.0013
13	0.0369	0.0345	0.0127	-0.0045	-0.0100	0.0020
14	0.0334	0.0229	0.0005	-0.0091	-0.0053	0.0019
15	0.0257	0.0114	-0.0060	-0.0083	-0.0011	0.0008
16	0.0167	0.0024	-0.0079	-0.0052	0.0012	0.0000
17	0.0083	-0.0034	-0.0067	-0.0020	0.0017	-0.0003
18	0.0017	-0.0060	-0.0043	0.0003	0.0012	-0.0002
19	-0.0027	-0.0063	-0.0018	0.0013	0.0005	-0.0001

20	-0.0049	-0.0051	0.0000	0.0014	0.0000	...
21	-0.0055	-0.0034	0.0010	0.0010	-0.0002	...
22	-0.0049	-0.0017	0.0012	0.0005	-0.0002	...
23	-0.0037	-0.0003	0.0010	0.0001	-0.0001	...
24	-0.0023	0.0006	0.0007	-0.0012	...	...
25	-0.0010	0.0010	0.0003	-0.0002	...	...
26	-0.0001	0.0010	0.0000	-0.0002	...	...
27	0.0005	0.0008	-0.0001	-0.0001	...	...
28	0.0008	0.0005	-0.0002	...	...	...
29	0.0009	0.0003	-0.0002	...	...	...
30	0.0007	0.0000	-0.0001	...	...	...
31	0.0005	-0.0001	...	...	...	...
32	0.0003	-0.0002	...	...	...	...
33	0.0001	-0.0002	...	...	...	...
34	0.0000	-0.0001	...	...	...	...
35	-0.0001	-0.0001	...	...	...	...
36	-0.0001	...	...	...	...	...
37	-0.0001	...	...	...	...	...
38	-0.0001	...	...	...	...	...
39	-0.0001	...	...	...	...	...
40	...	...	...	...	...	...

6.0 PROTOTYPE ALGORITHM RESULTS

This section provides an overview of the results obtained from a prototype UNIX FORTRAN F77 (i.e., MICROWAY, Inc., NDP FORTRAN-386 running under MICROPORT'S - SYSTEM V/386 Unix Operating System). This program implements the probabilistic signal reconstruction formulations outlined in Sections 3.0 and 5.0. Appendix A, contains the main statistical nromal probability filtering (SNPF) program and supporting normal error probability statistical (NEPS.F) subroutine implementations used to produce these results. This prototype software is currently configured to consider only the one minute (coherently integrated) outputs from the (software) TYCHO Technology Inc. System. Hence no concensus averaging over time is possible under this software implementation.

6.1 GROUND CLUTTER ASSESSMENT

The first increment of data results consider the first twenty-five range gates for beam 1, extracted from the archived data file 14130612.90P. These lower gate echelons were selected to analyze the data filtering effects resulting from local ground clutter and non-uniform ground heating mechanisms. Since each of these lower range gates have both their raw and filtered dominant power spectral peaks near the central frequency, the emphasis for the data comparison was shifted to a collation of second largest peaks. To aid in the location of these secondary peaks, a triangle is placed at the filtered peak location in Figures 6.1-1 through 6.1-24. A summary of the results contained in these Figures is provided in Table 6-1.

FIGURE 6.1-1 RANGE GATE 2 of BEAM 1 DEVELOPED FROM RANGE GATES 1-3

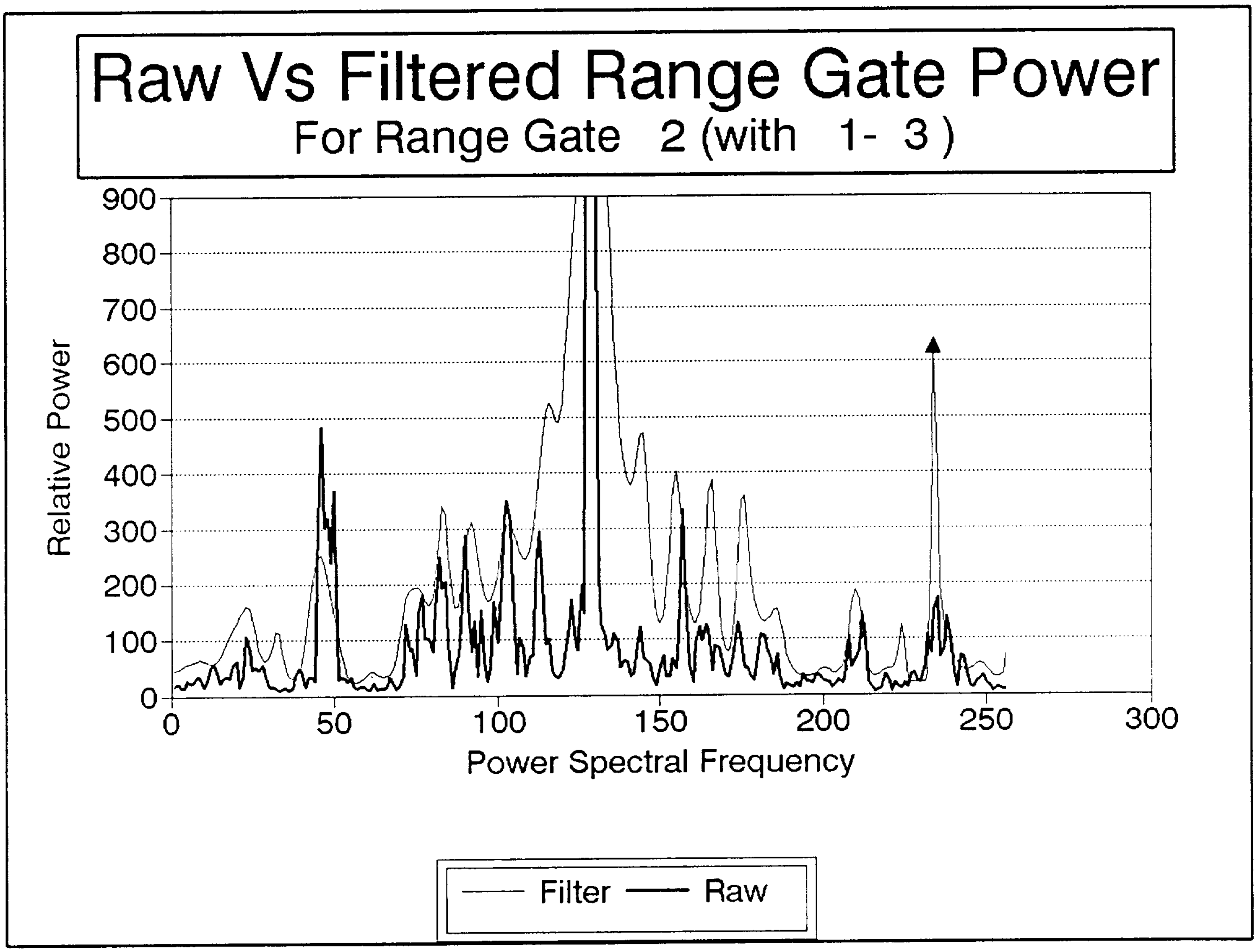


FIGURE 6.1-2 RANGE GATE 3 OF BEAM 1 DEVELOPED FROM RANGE GATES 2-4

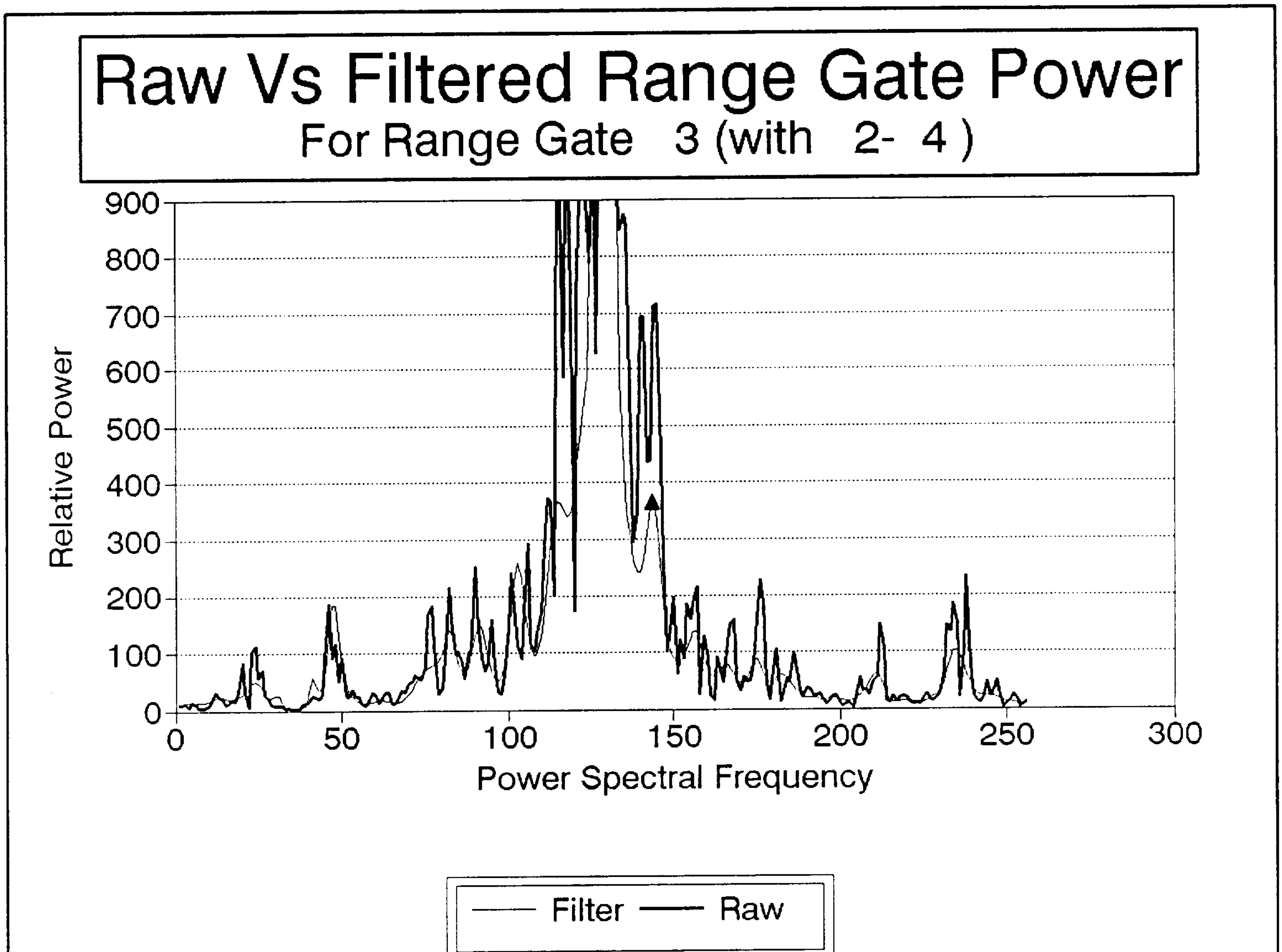


FIGURE 6.1-3 RANGE GATE 4 OF BEAM 1 DEVELOPED FROM RANGE GATES 3-5

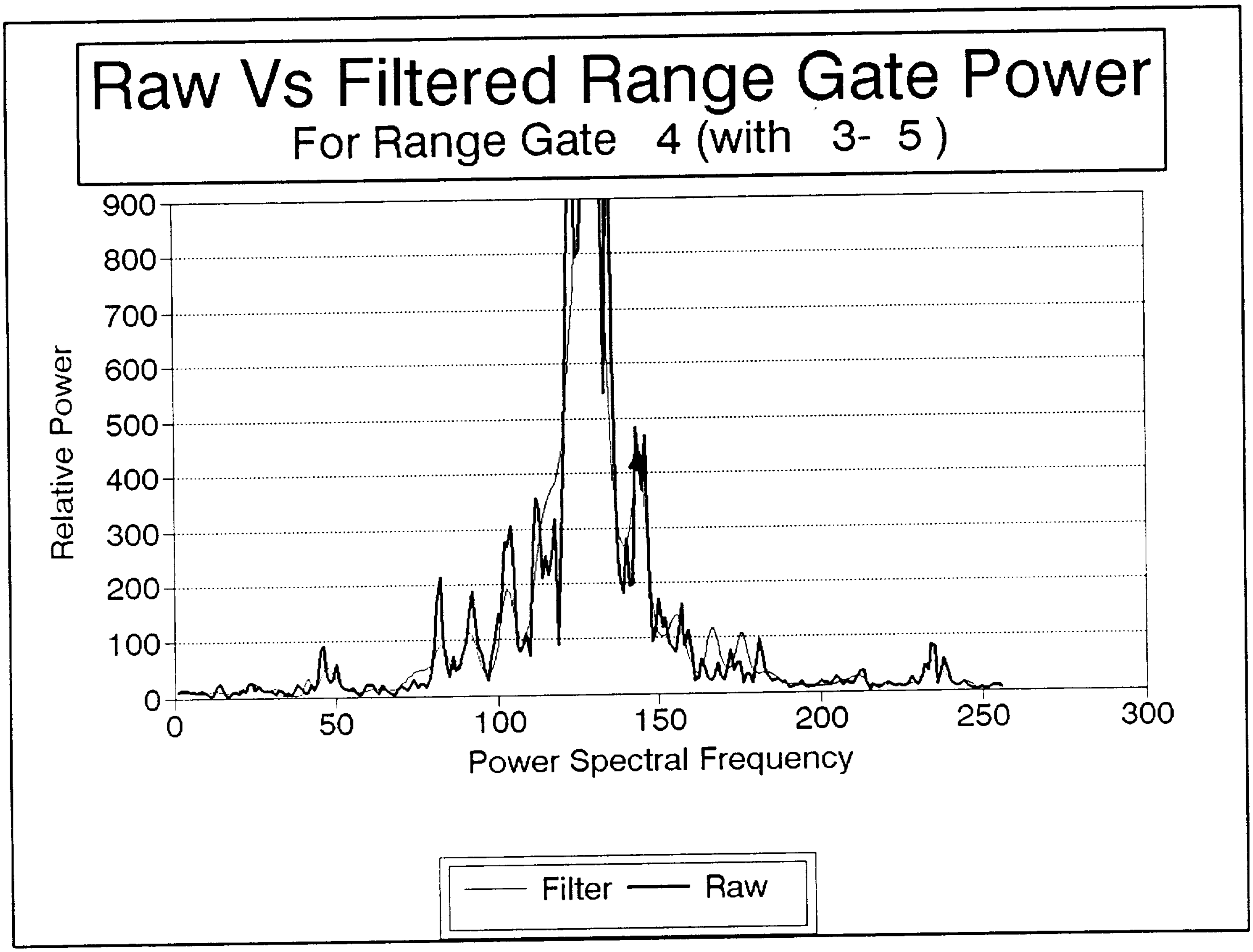


FIGURE 6.1-4 RANGE GATE 5 OF BEAM 1 DEVELOPED FROM RANGE GATES 4-6

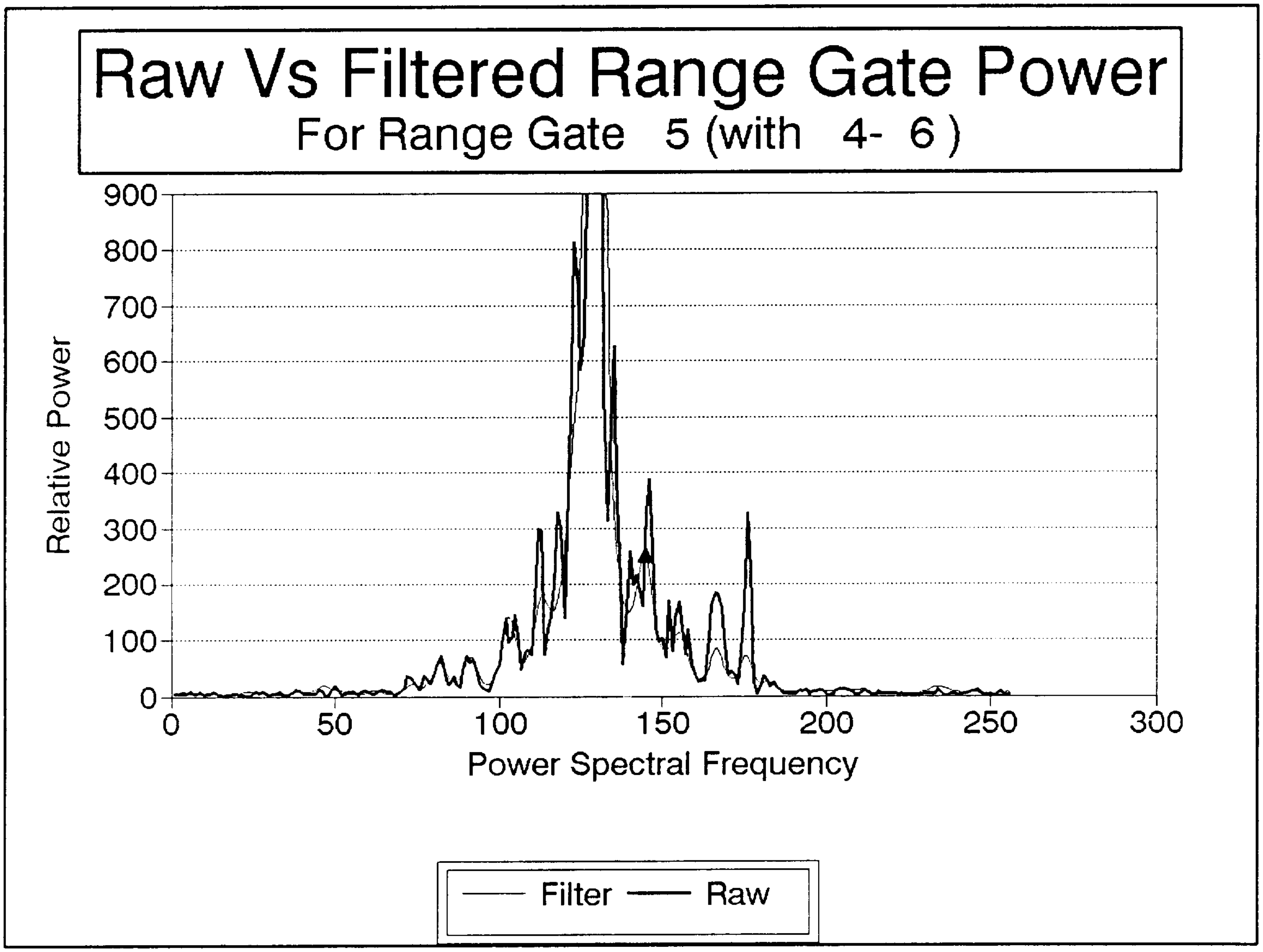


FIGURE 6.1-5 RANGE GATE 6 OF BEAM 1 DEVELOPED FROM RANGE GATES 5-7

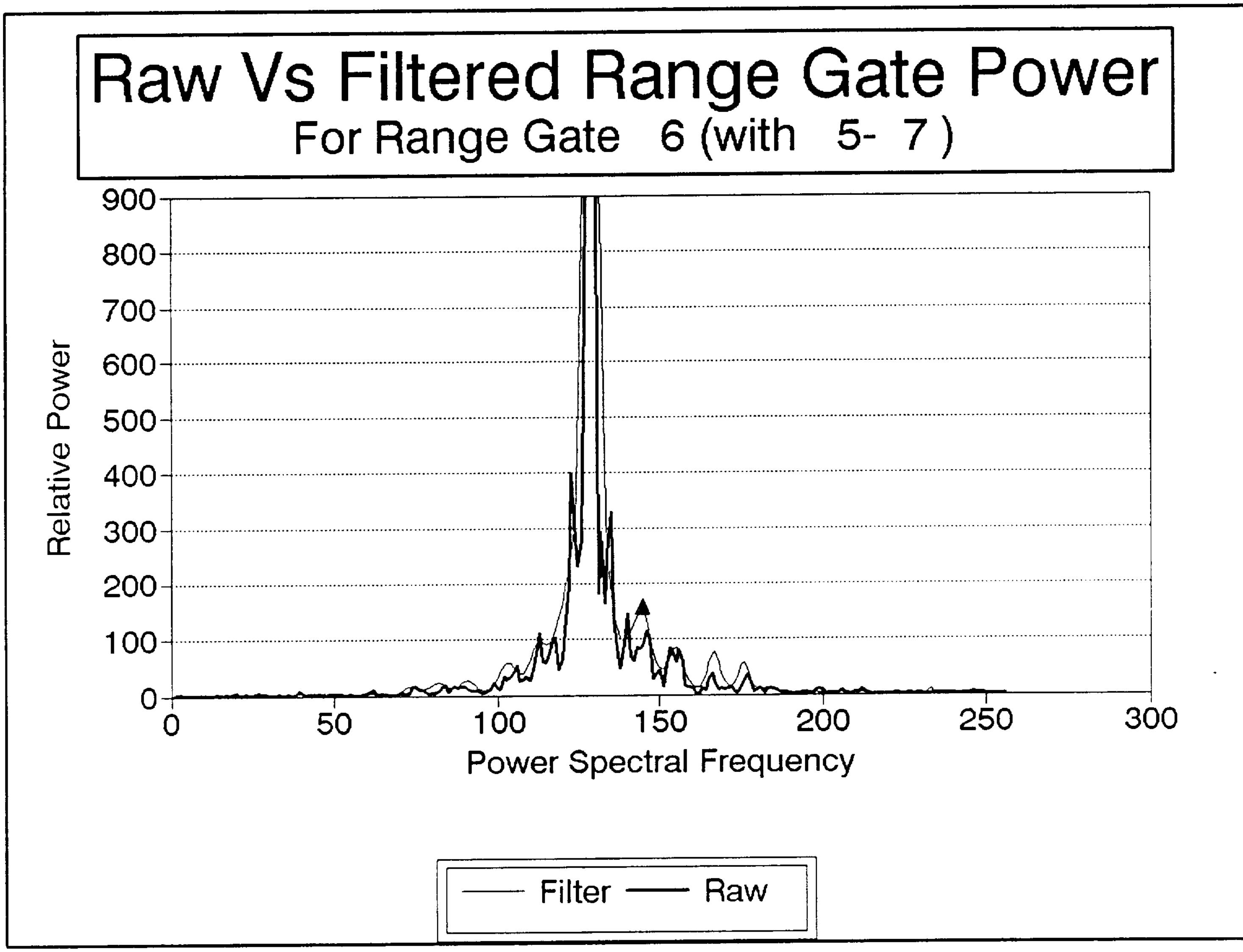


FIGURE 6.1-6 RANGE GATE 7 OF BEAM 1 DEVELOPED FROM RANGE GATES 6-8

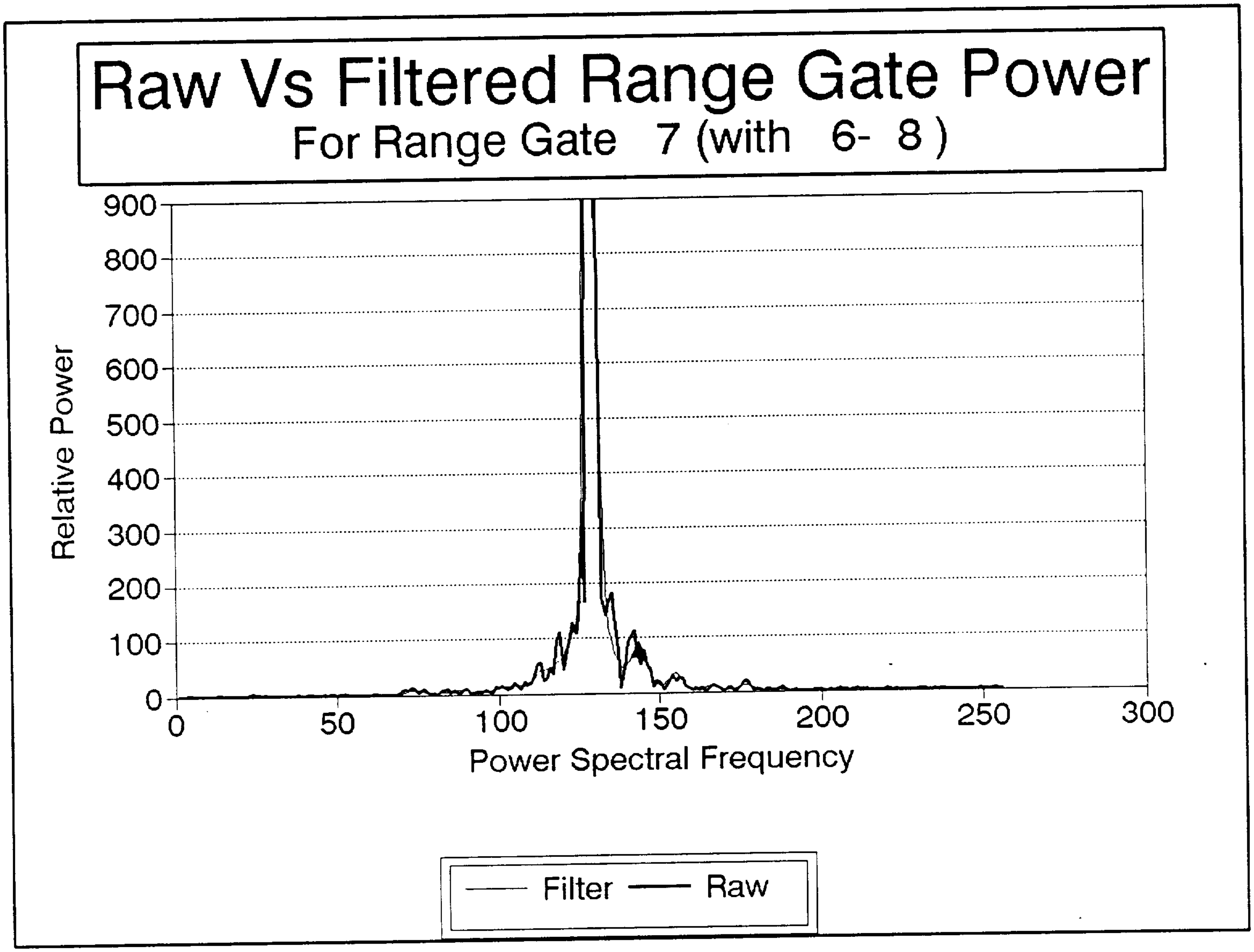


FIGURE 6.1-7 RANGE GATE 8 OF BEAM 1 DEVELOPED FROM RANGE GATES 7-9

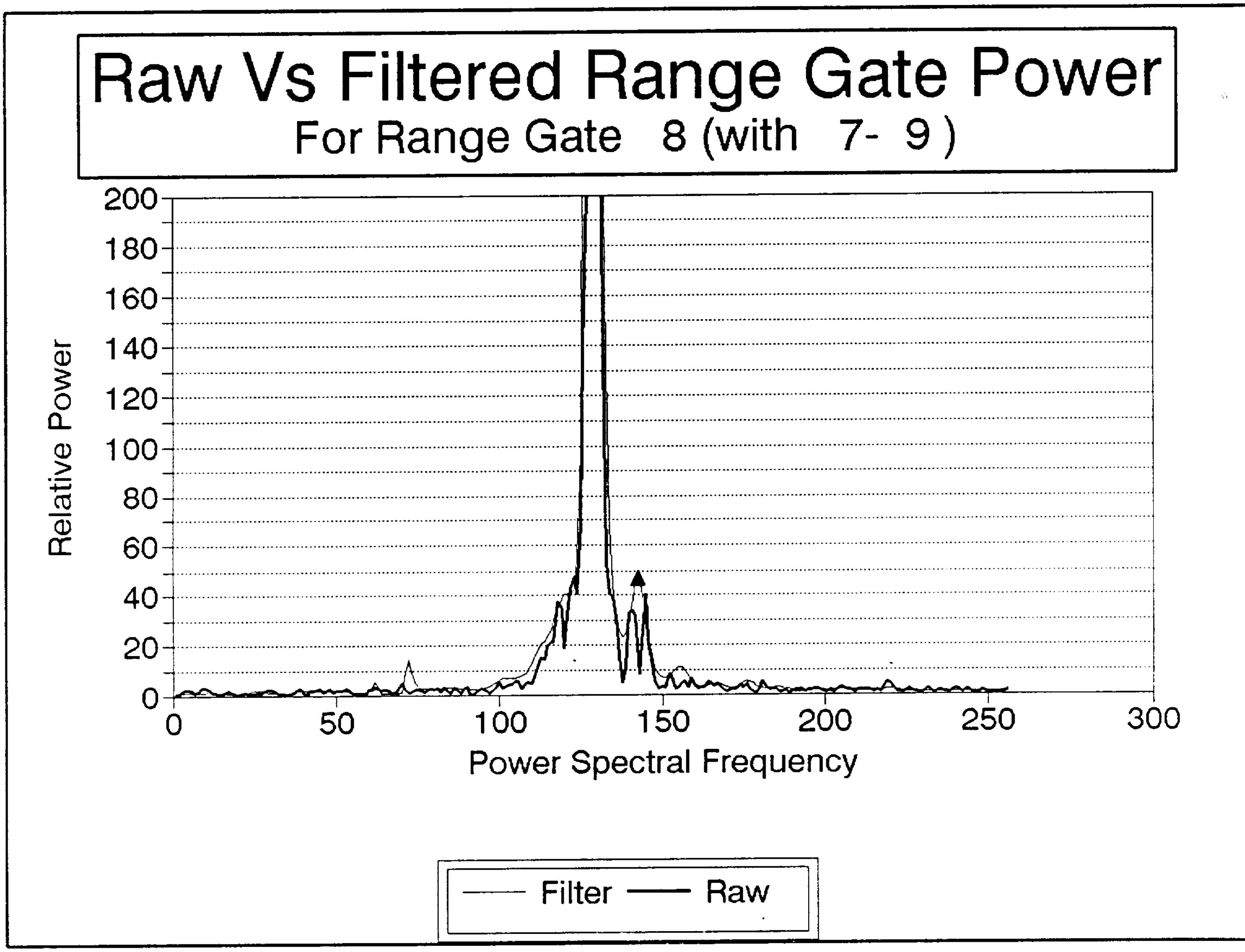


FIGURE 6.1-8 RANGE GATE 9 OF BEAM 1 DEVELOPED FROM RANGE GATES 8-10

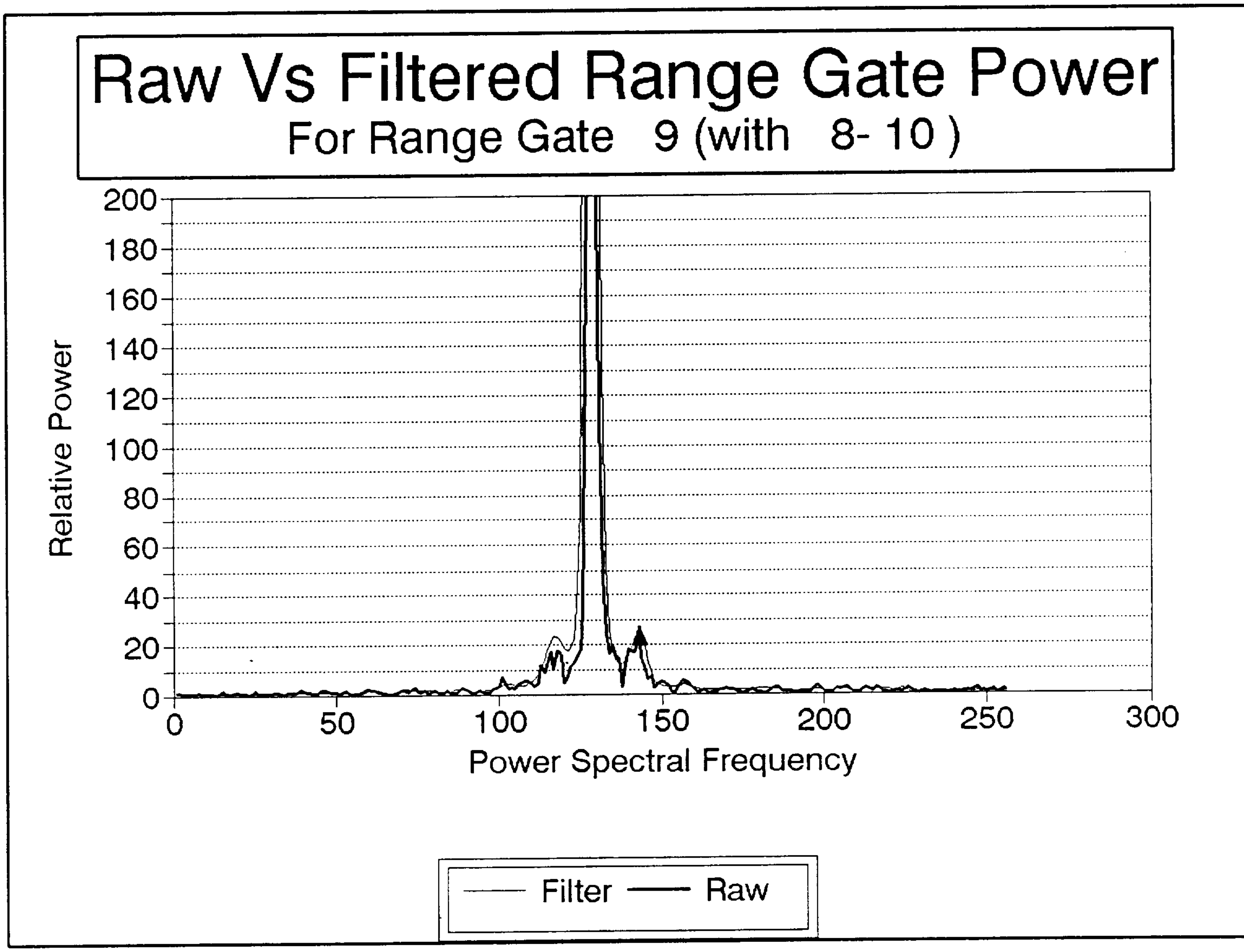


FIGURE 6.1-9 RANGE GATE 10 OF BEAM 1 DEVELOPED FROM RANGE GATES 9-11

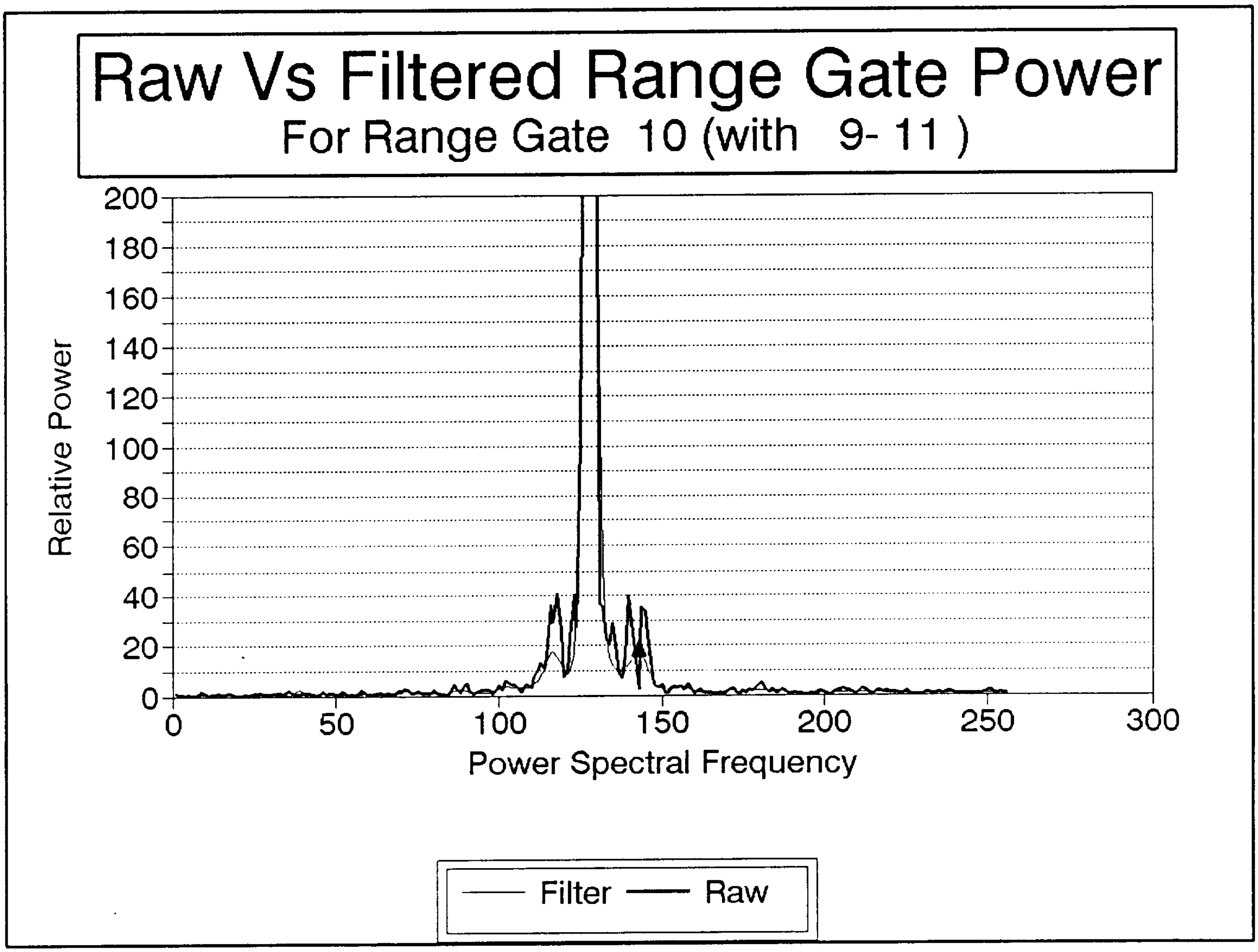


FIGURE 6.1-10 RANGE GATE 11 OF BEAM 1 DEVELOPED FROM RANGE GATES 10-12

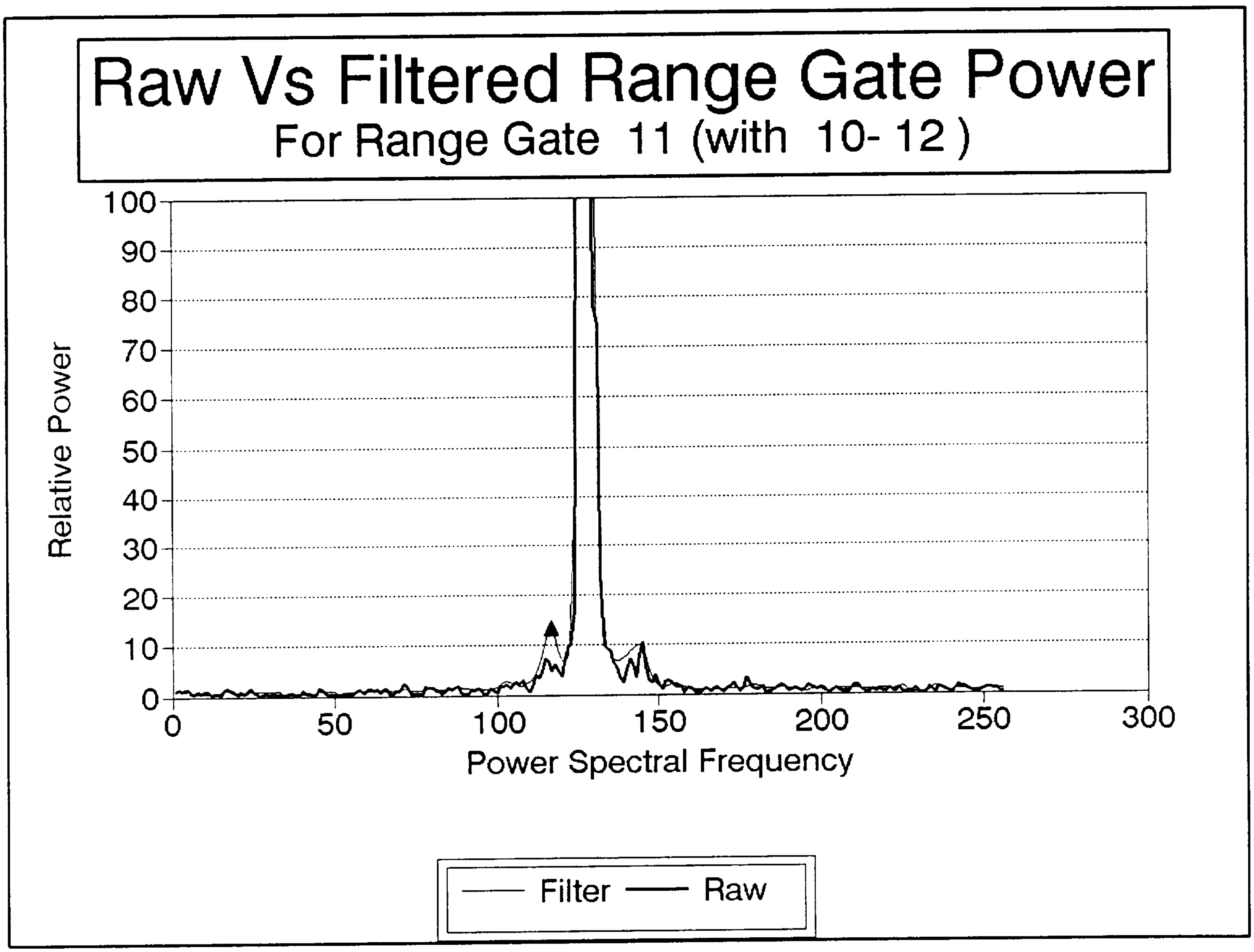


FIGURE 6.1-11 RANGE GATE 12 OF BEAM 1 DEVELOPED FROM RANGE GATES 11-13

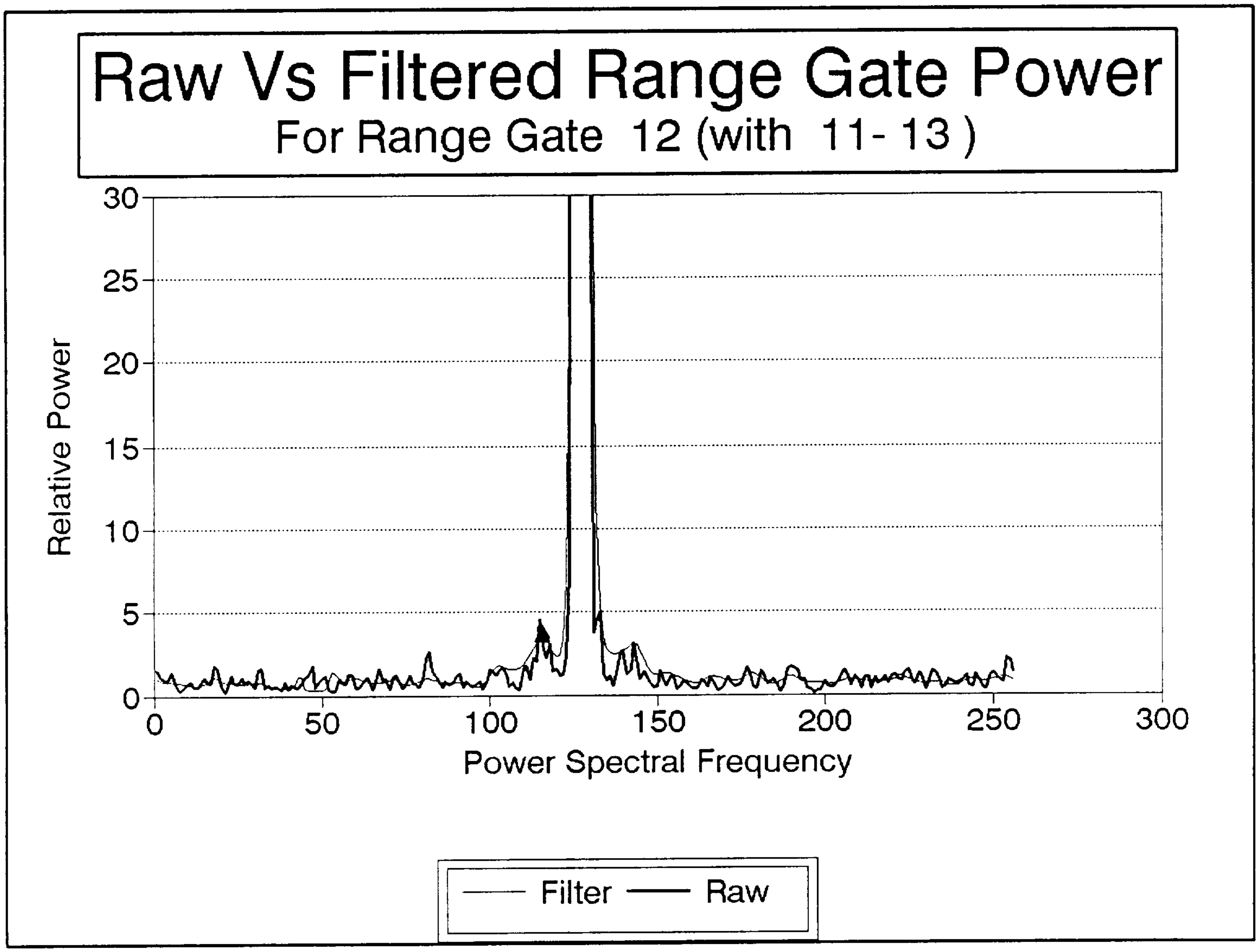


FIGURE 6.1-12 RANGE GATE 13 OF BEAM 1 DEVELOPED FROM RANGE GATES 12-14

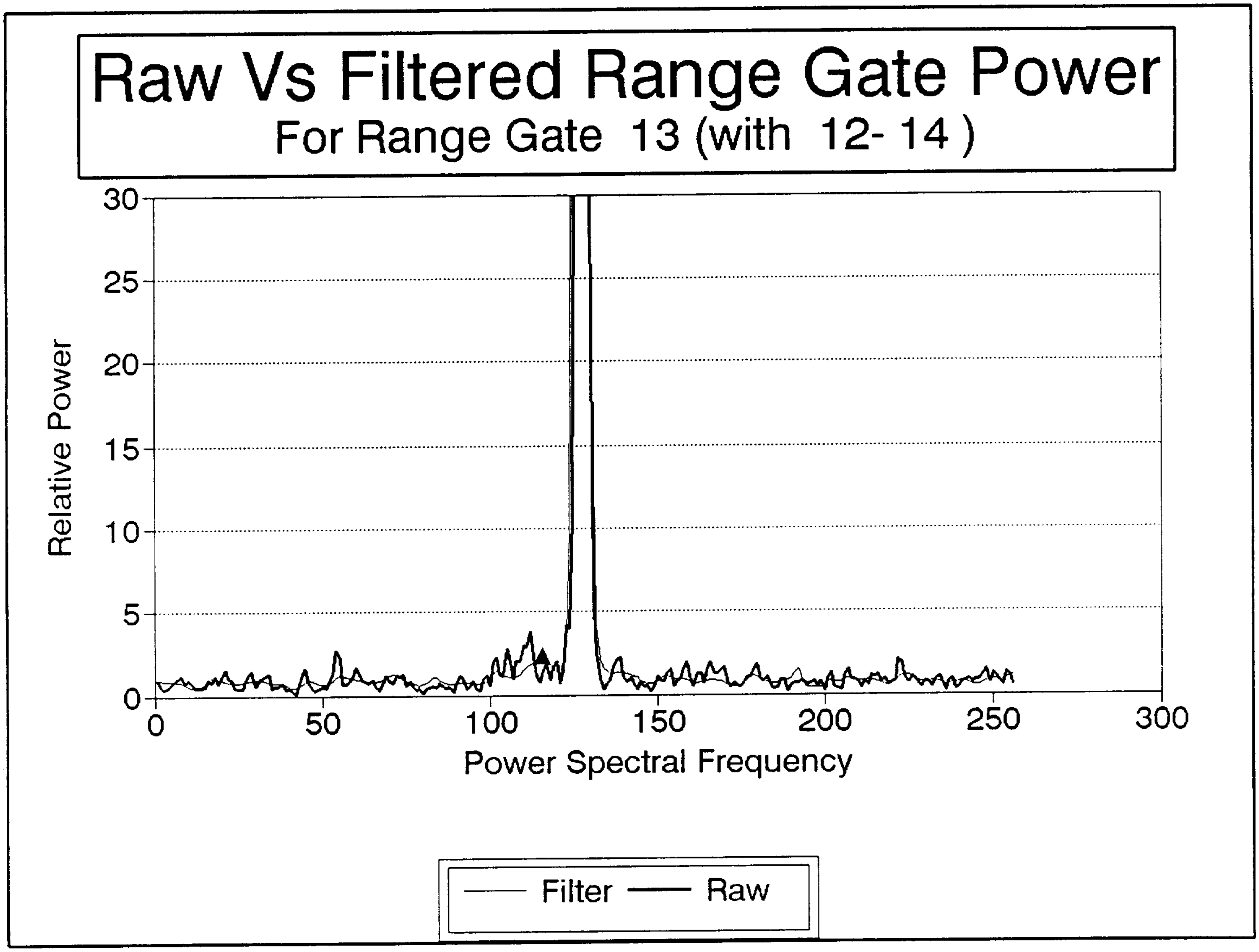


FIGURE 6.1-13 RANGE GATE 14 OF BEAM 1 DEVELOPED FROM RANGE GATES 13-15

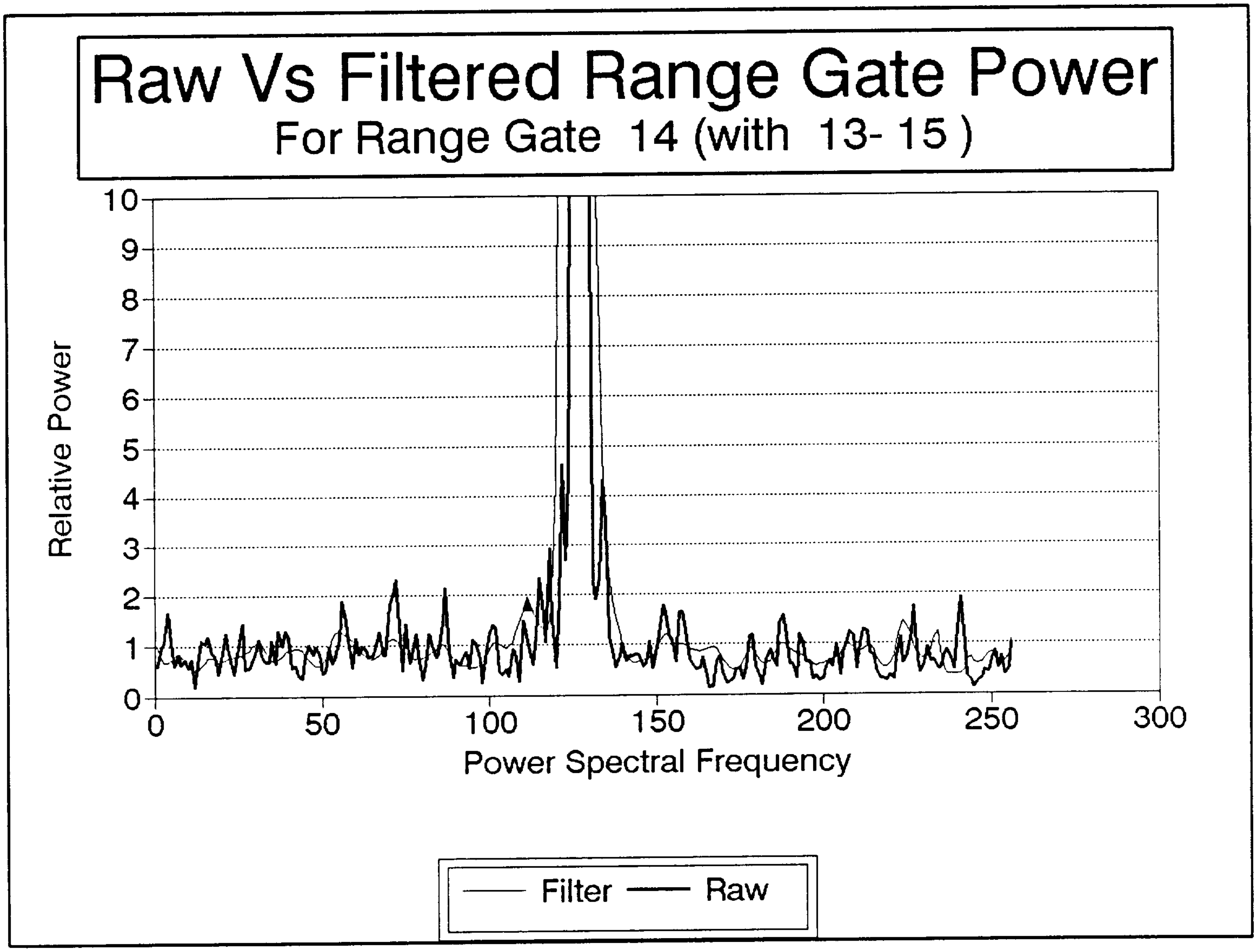


FIGURE 6.1-14 RANGE GATE 15 OF BEAM 1 DEVELOPED FROM RANGE GATES 14-16

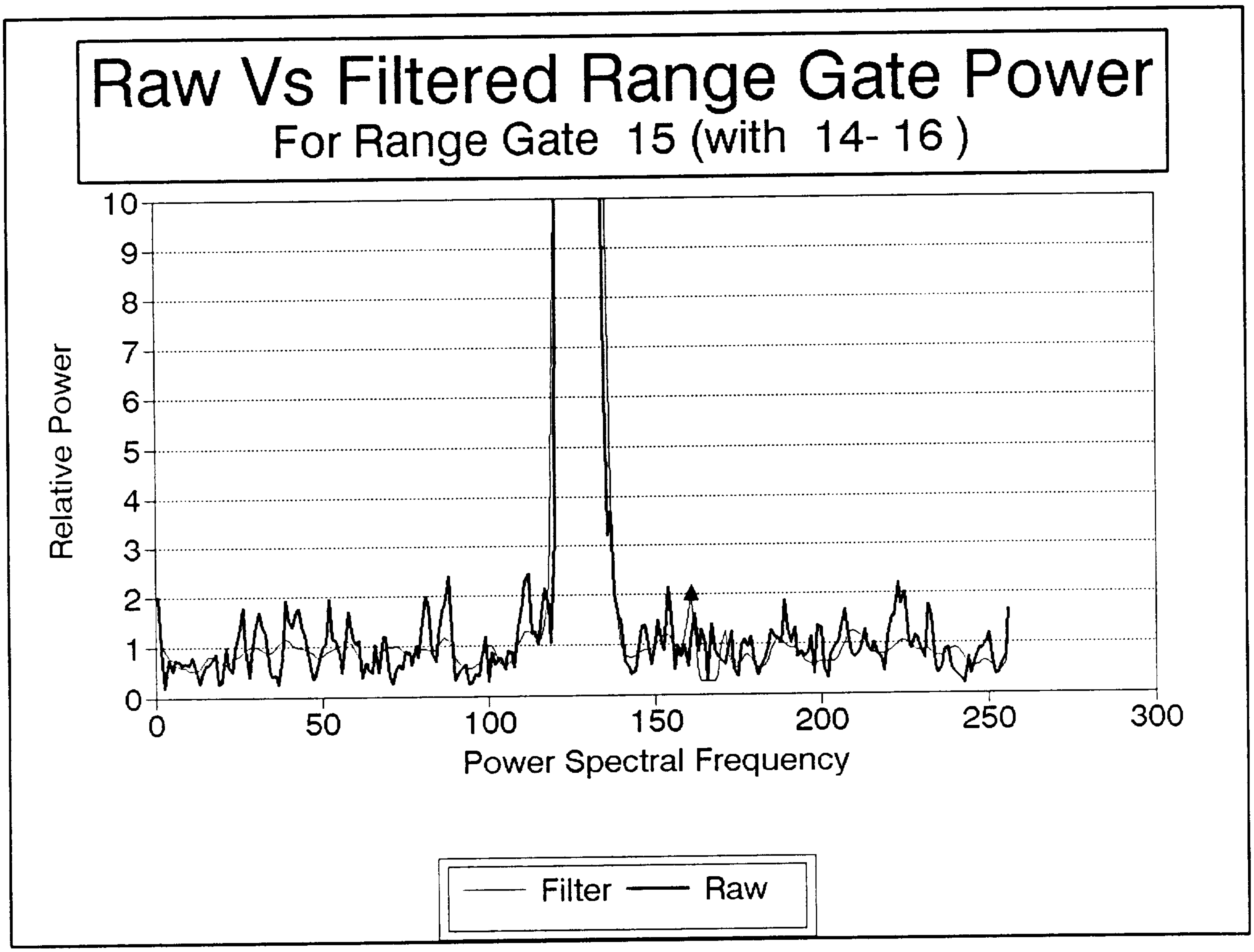


FIGURE 6.1-15 RANGE GATE 16 OF BEAM 1 DEVELOPED FROM RANGE GATES 15-17

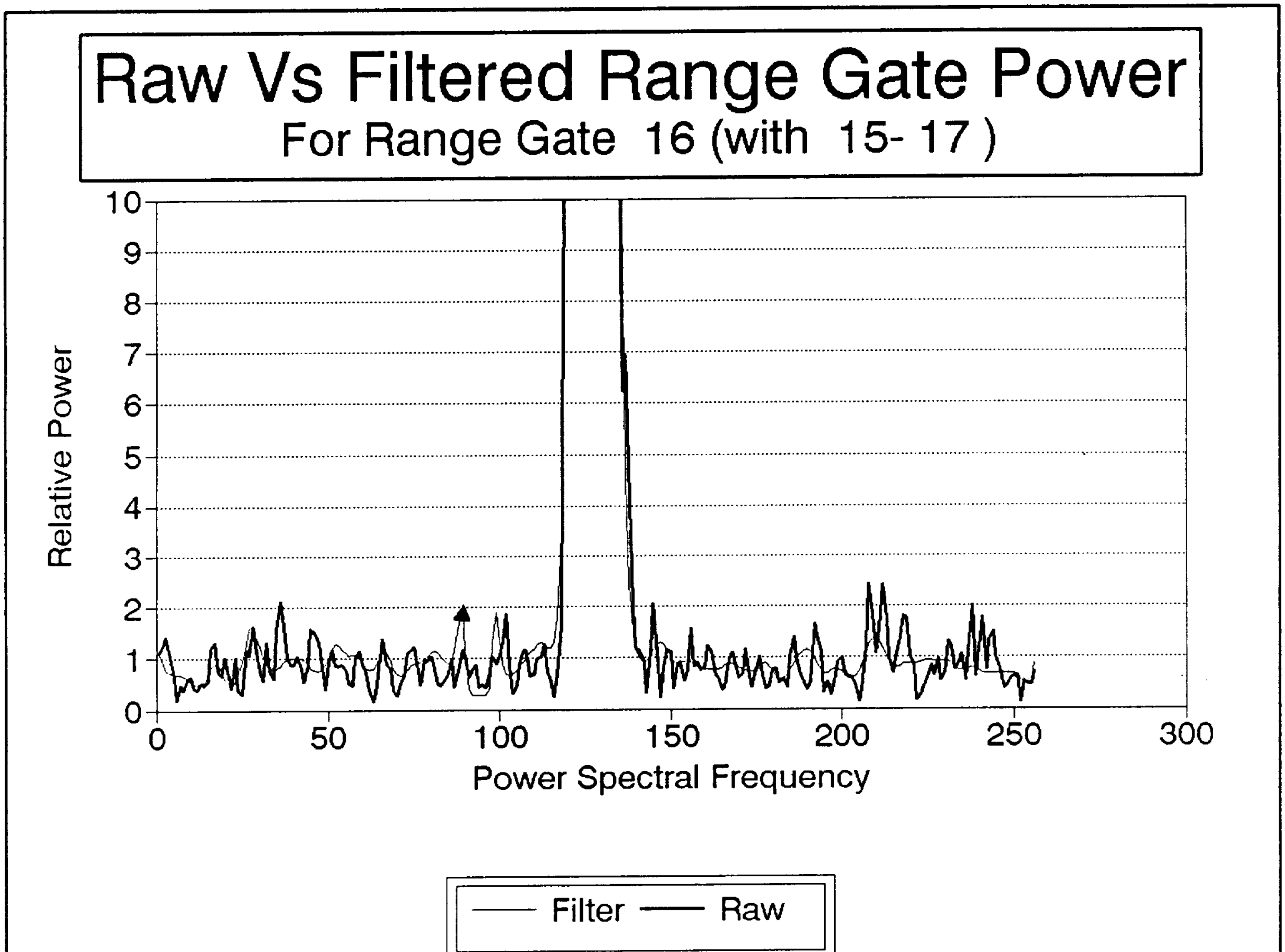


FIGURE 6.1-16 RANGE GATE 17 OF BEAM 1 DEVELOPED FROM RANGE GATES 16-18

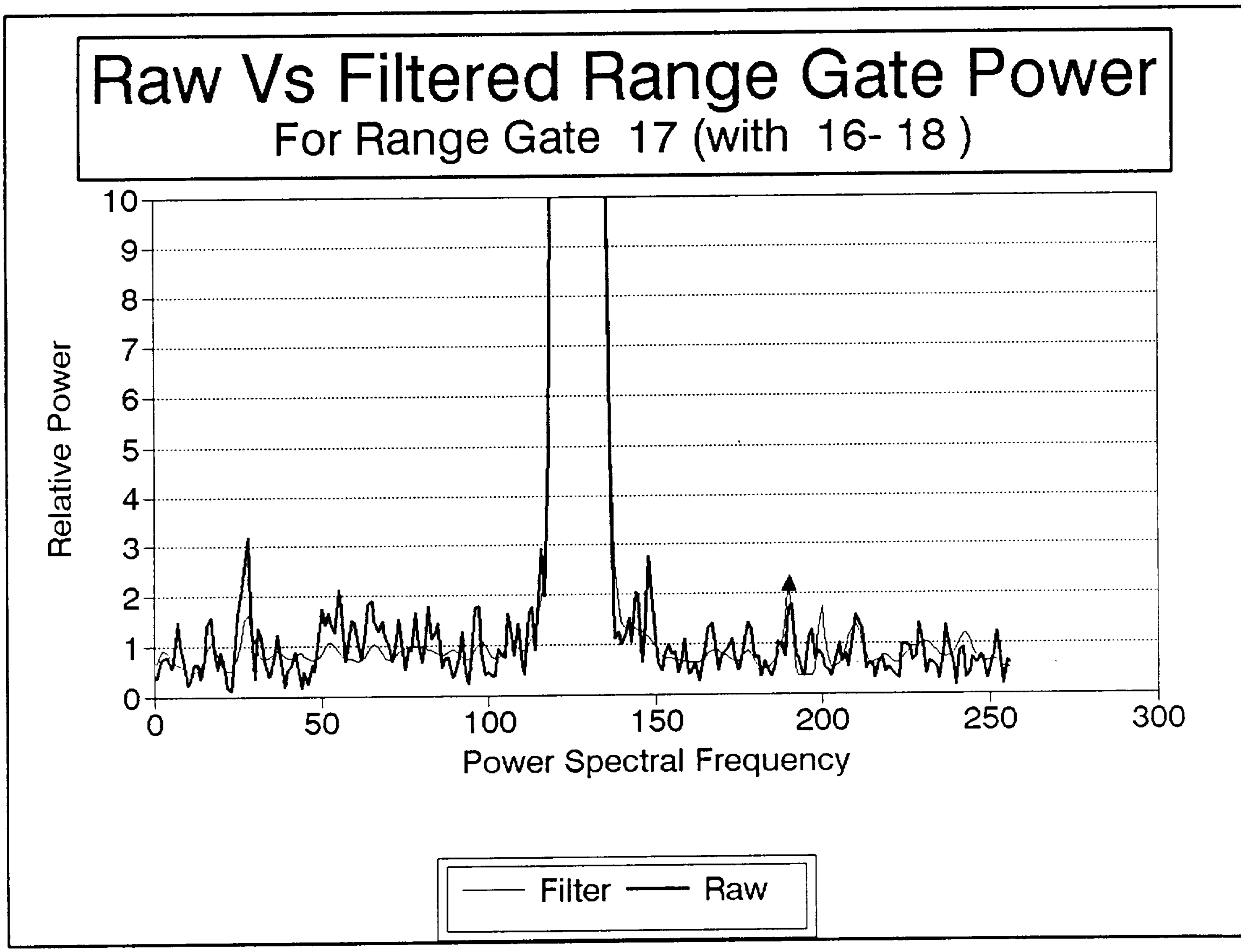


FIGURE 6.1-17 RANGE GATE 18 OF BEAM 1 DEVELOPED FROM RANGE GATES 17-19

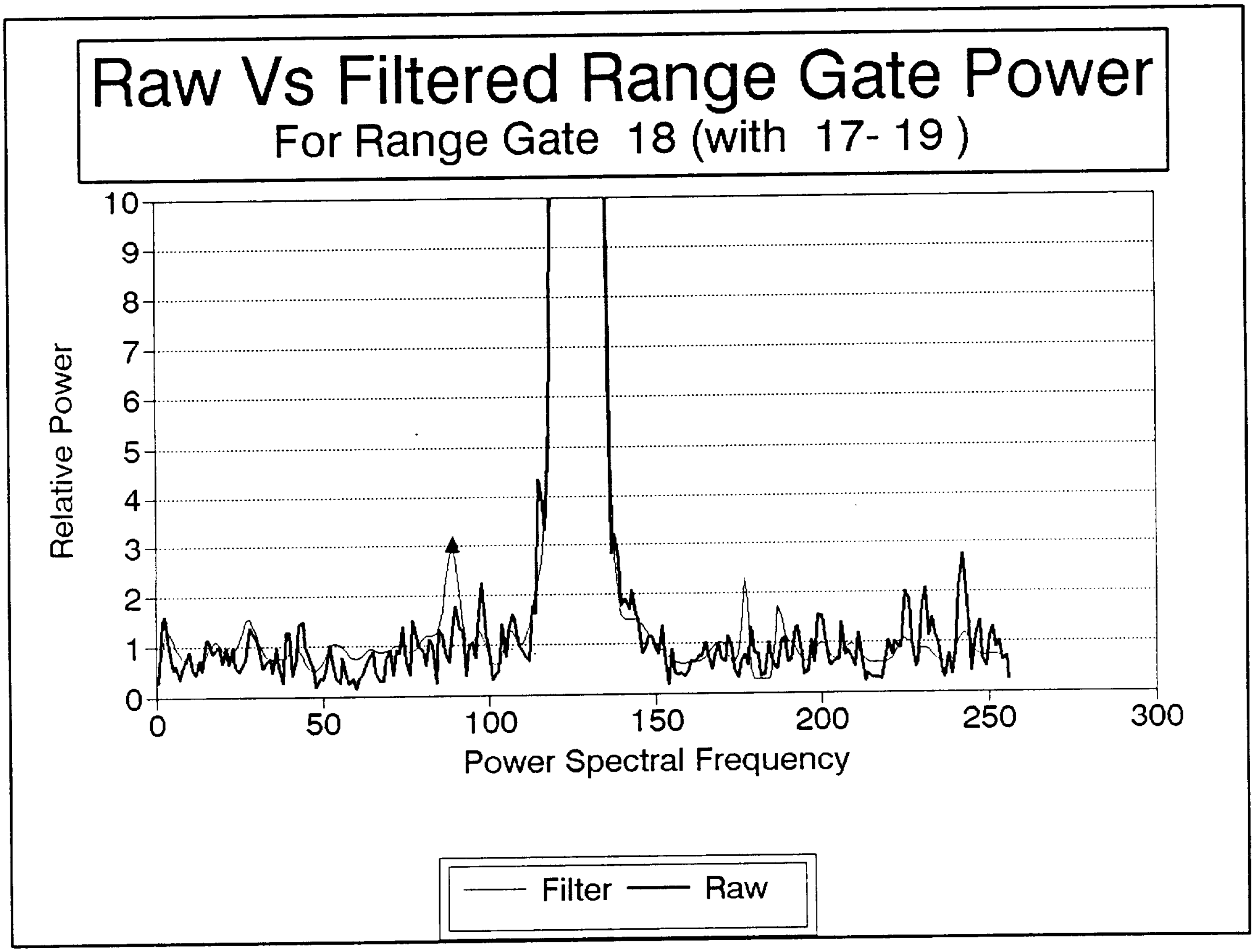


FIGURE 6.1-18   RANGE GATE 19 OF BEAM 1 DEVELOPED FROM RANGE GATES 18-20

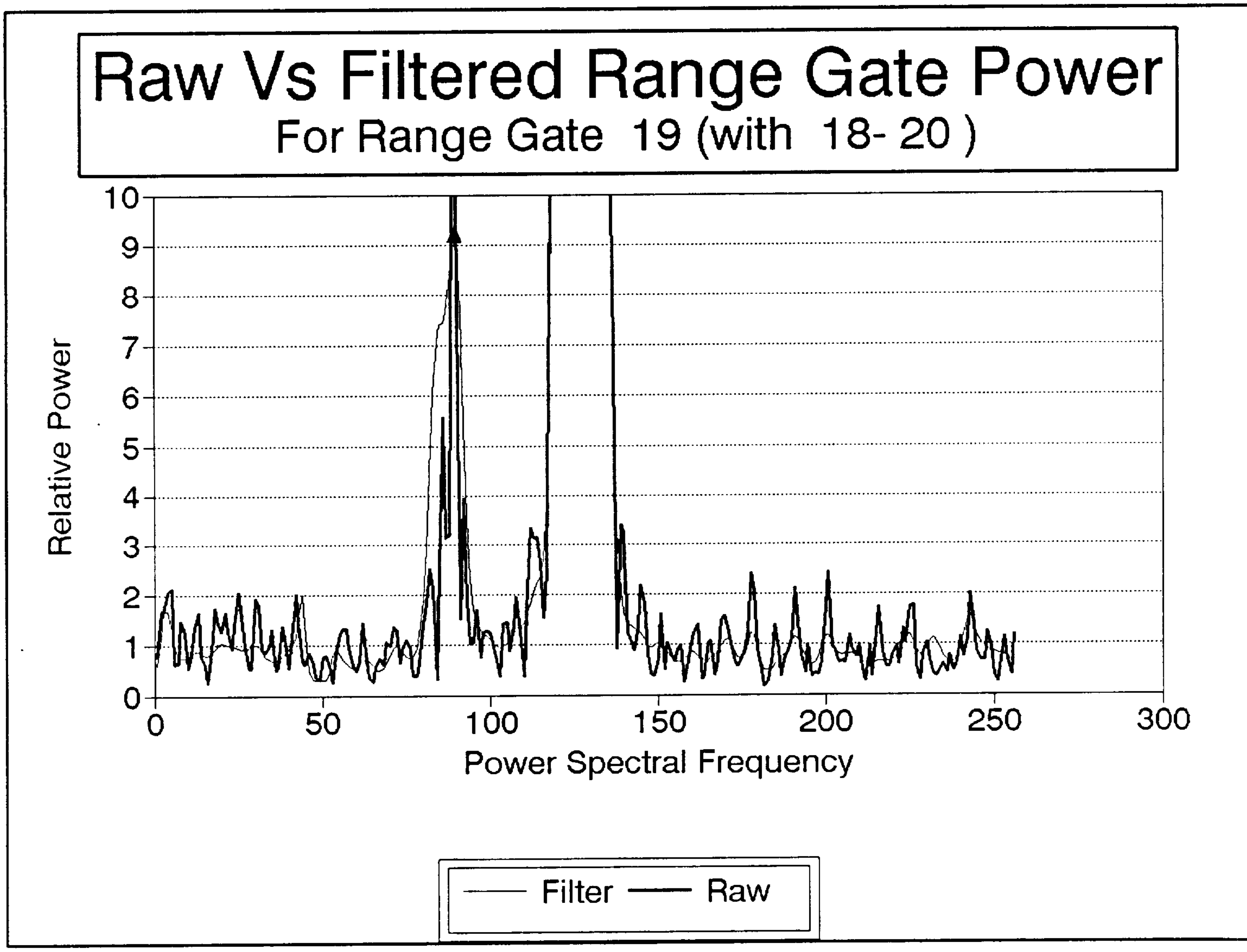


FIGURE 6.1-19 RANGE GATE 20 OF BEAM 1 DEVELOPED FROM RANGE GATES 19-21

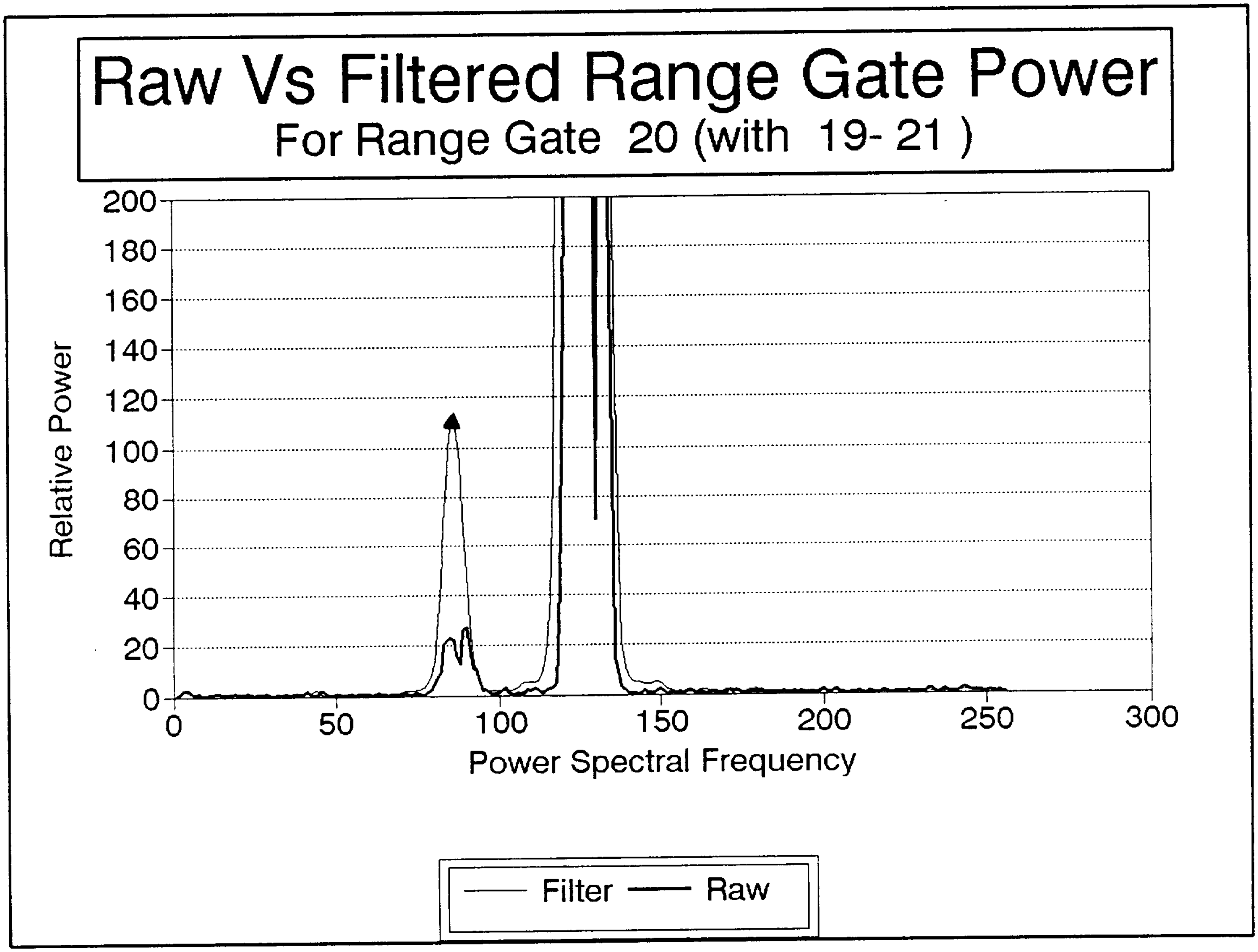


FIGURE 6.1-20 RANGE GATE 21 OF BEAM 1 DEVELOPED FROM RANGE GATES 20-22

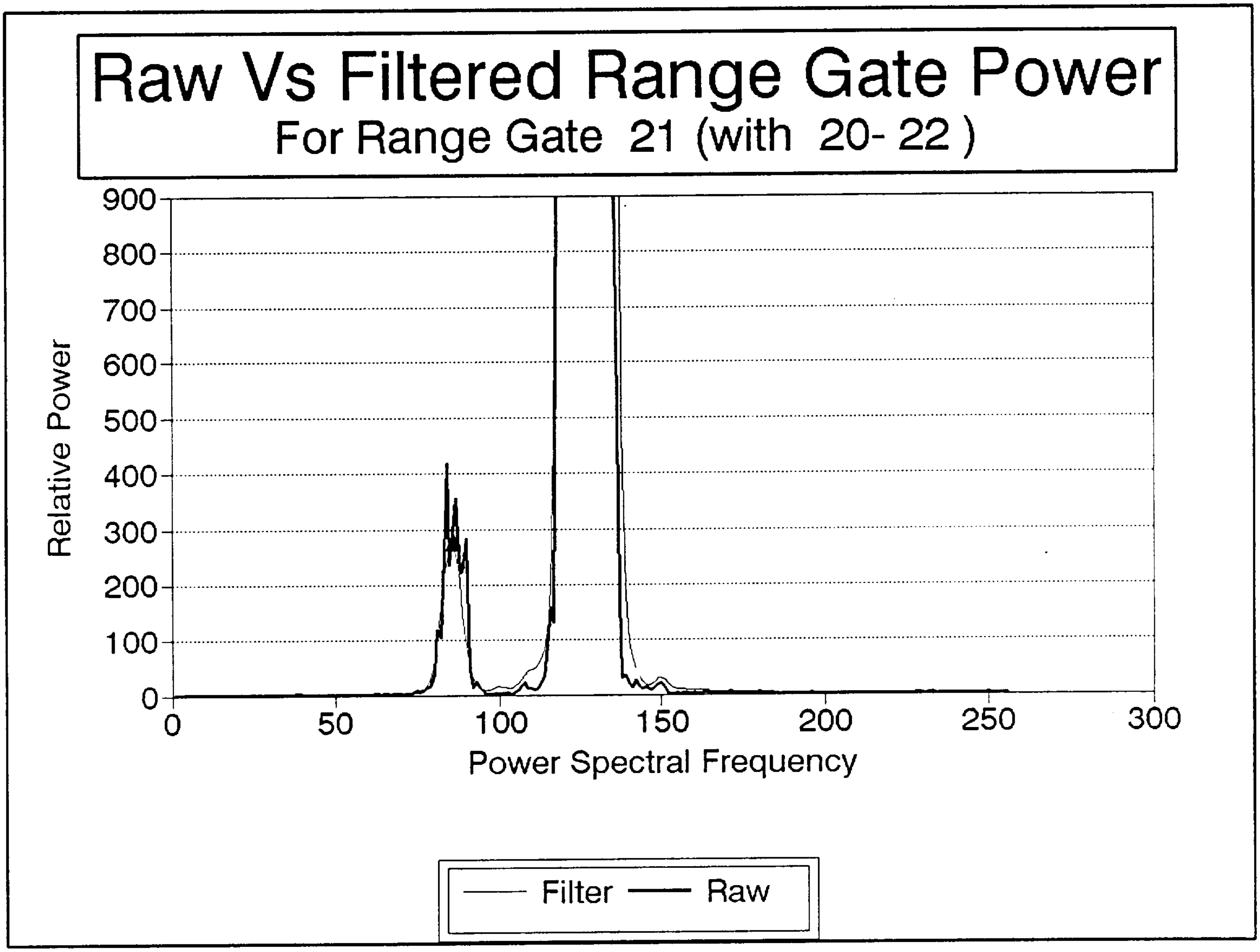


FIGURE 6.1-21 RANGE GATE 22 OF BEAM 1 DEVELOPED FROM RANGE GATES 21-23

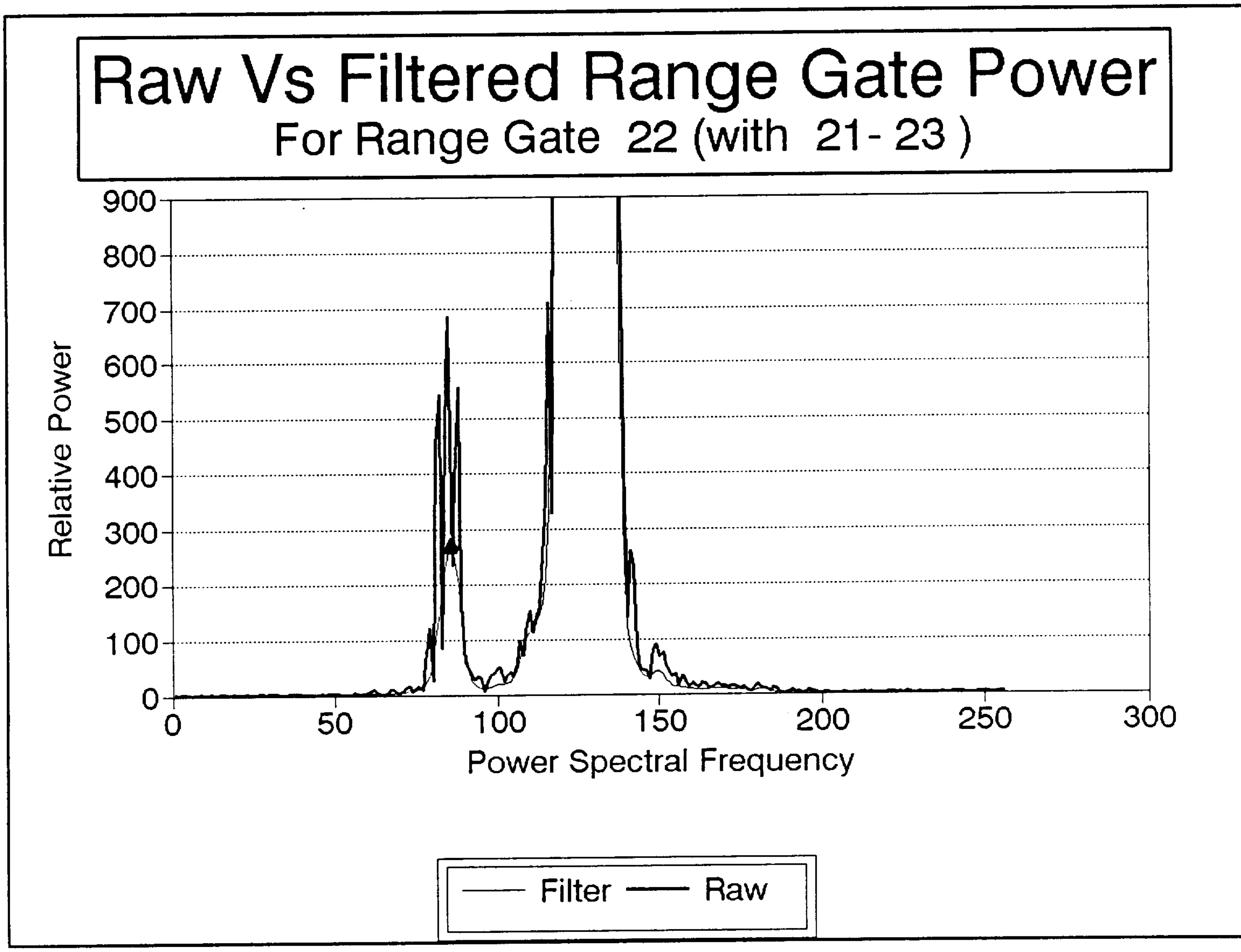


FIGURE 6.1-22 RANGE GATE 23 OF BEAM 1 DEVELOPED FROM RANGE GATES 22-24

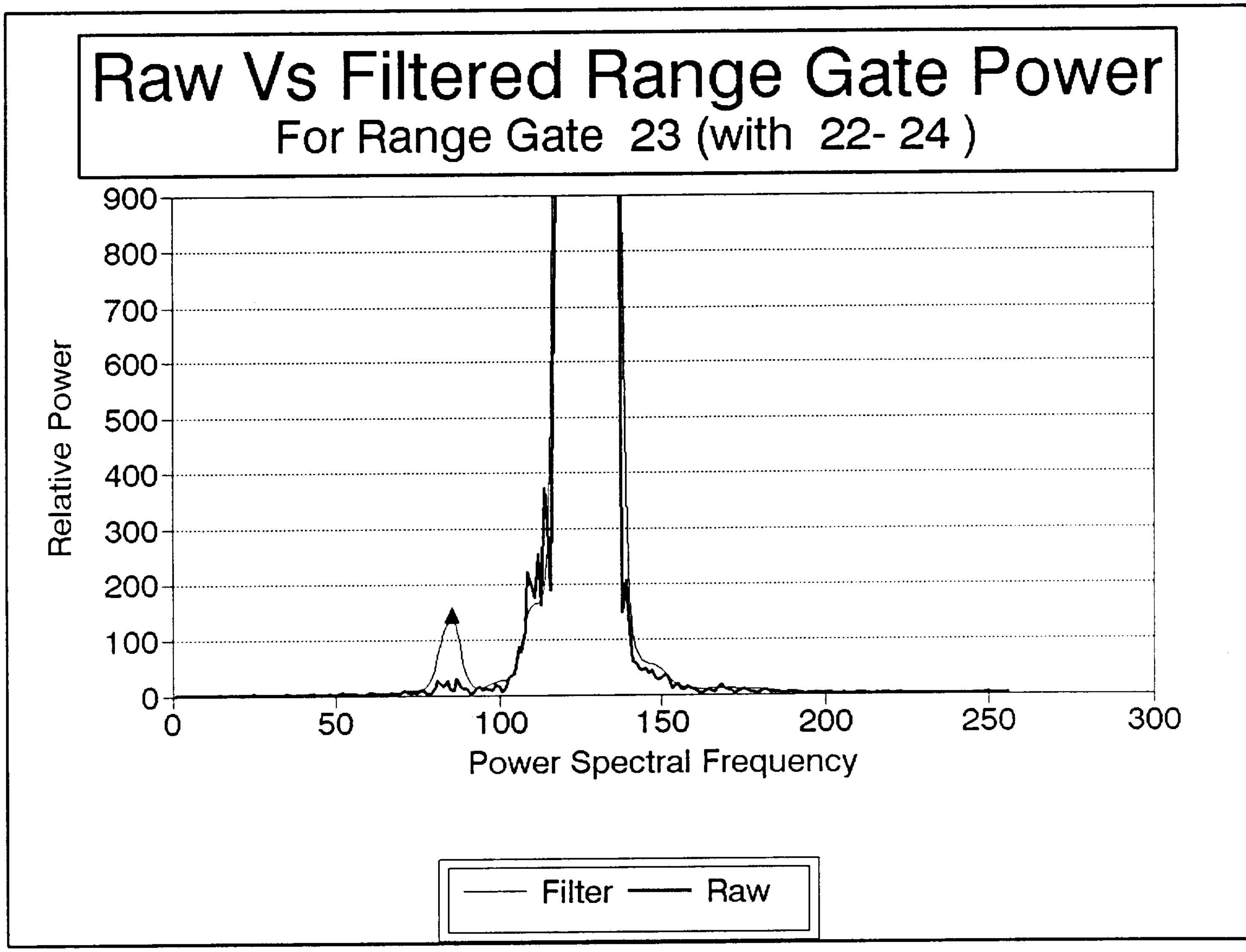


FIGURE 6.1-23 RANGE GATE 24 OF BEAM 1 DEVELOPED FROM RANGE GATES 23-25

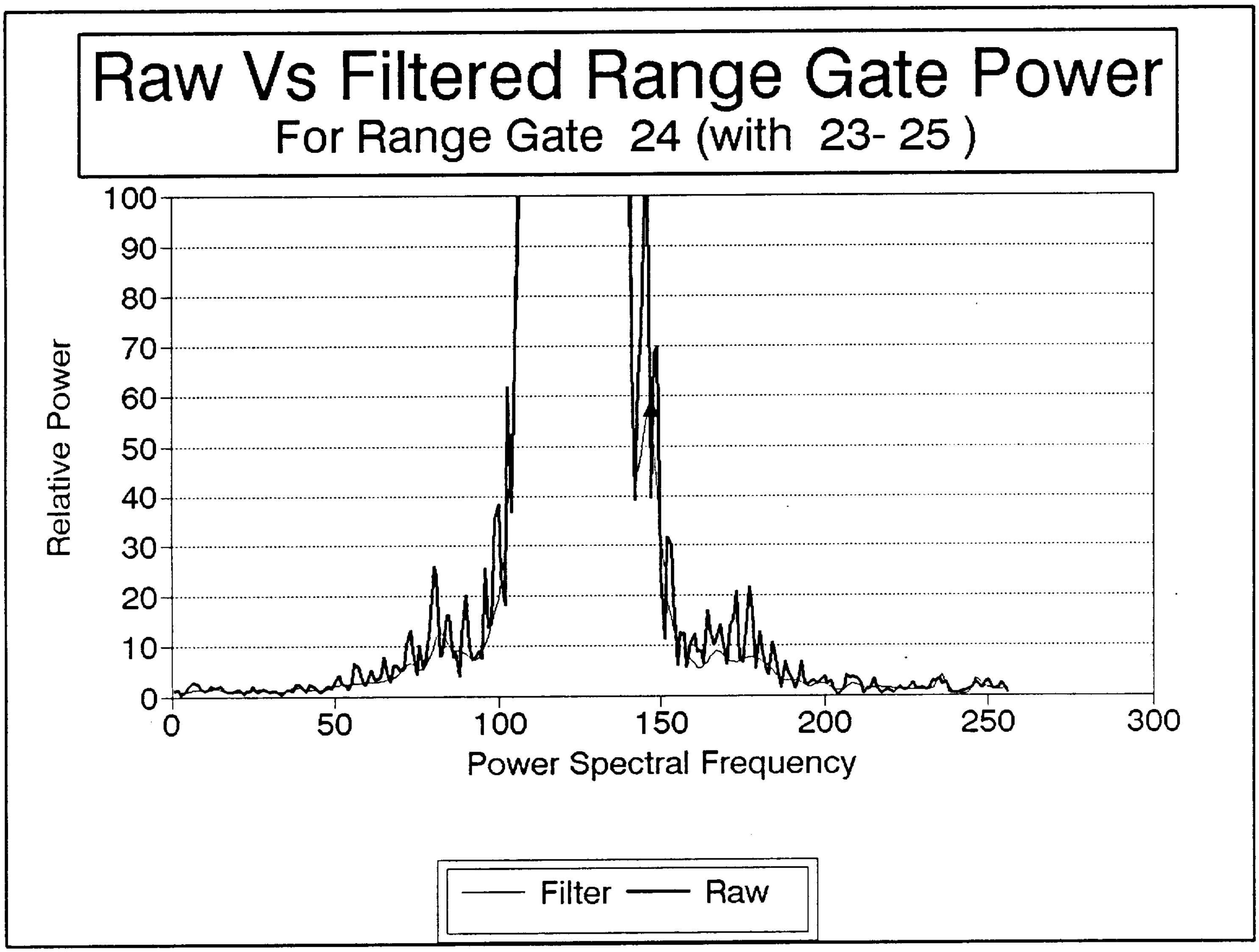
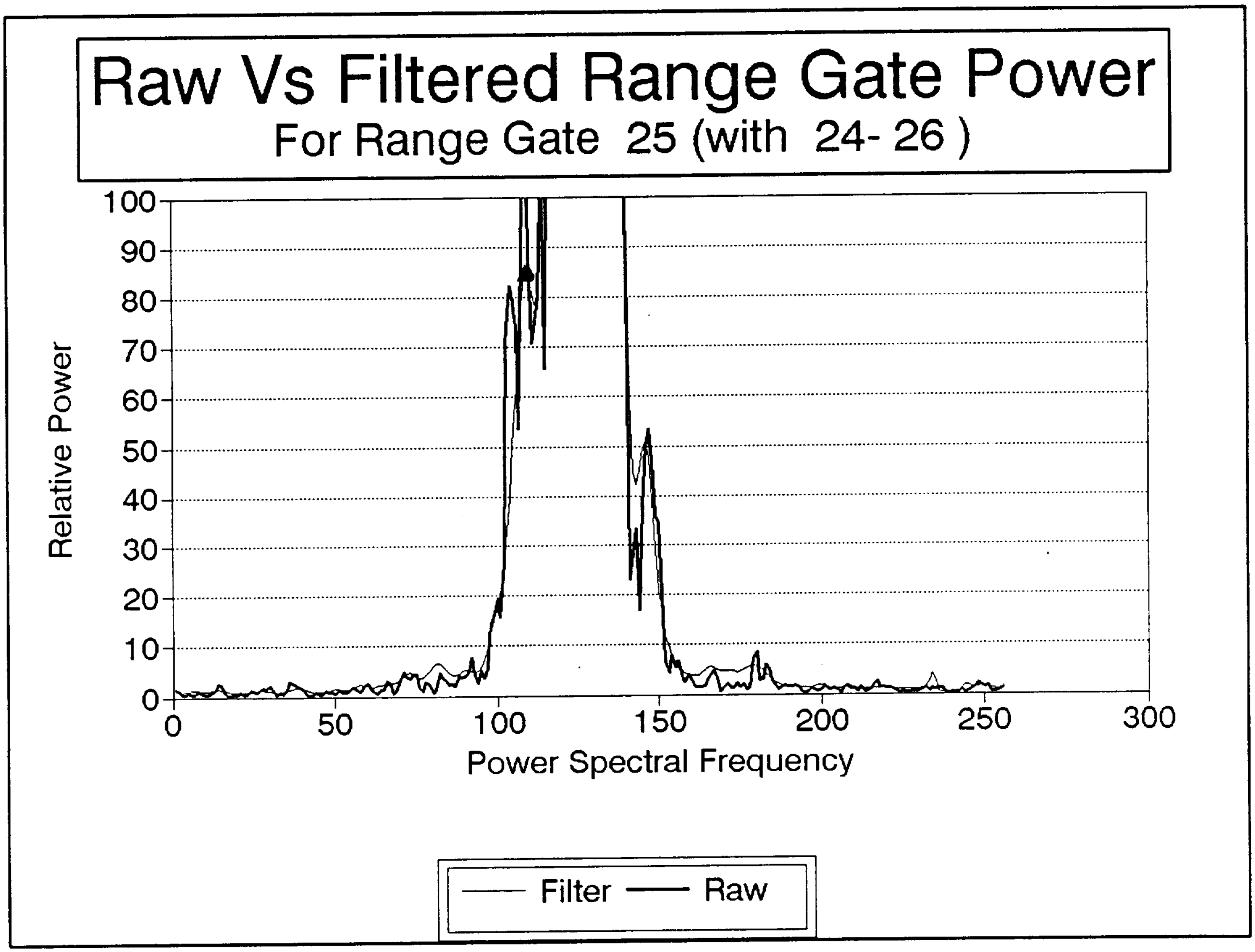


FIGURE 6.1-24 RANGE GATE 25 OF BEAM 1 DEVELOPED FROM RANGE GATES 24-26



Given the variability in frequency bin locations and raw data variance of Table 6-1, between range gates 9 through 19, a second increment of data was computed. This process used two range gates on each side of the desired range gate. These results are provided in Table 6-2.

TABLE 6-1
SUMMARIZED BEAM 1, LOWER RANGE GATE RESULTS WITH ONE SURROUNDING RANGE GATE

RANGE GATE NUMBER	FILTERED SECOND RELATIVE POWER AMPLITUDE	LARGEST PEAK FREQUENCY BIN	RAW DATA VARIANCE	FILTERED DATA VARIANCE
2	623.59	234	37.83	8.83
3	370.86	144	30.92	7.13
4	424.24	144	29.76	6.69
5	255.89	145	28.93	6.42
6	150.28	144	28.68	6.29
7	67.96	143	24.73	9.79
8	47.16	143	25.79	5.67
9	24.37	143	24.39	5.37
10	17.54	143	26.28	5.59
11	13.38	117	26.18	5.29
12	3.48	116	24.28	5.30
13	2.03	116	20.90	6.96
14	1.84	111	-	-
15	2.07	161	23.54	5.43
16	1.93	89	22.20	5.32
17	2.26	190	24.50	5.39
18	3.02	89	21.87	5.23
19	9.26	89	18.90	8.29
20	110.95	86	22.61	5.36
21	272.69	86	25.08	10.24
22	264.11	86	24.32	9.68
23	141.43	85	26.24	5.56
24	56.99	146	22.01	4.89
25	84.49	109	21.50	5.28

TABLE 6-2
RESULTS FROM BEAM 1 WITH TWO SURROUNDING RANGE GATES

RANGE GATE NUMBER	FILTERED RELATIVE POWER AMPLITUDE	LARGEST PEAK FREQUENCY BIN	RAW DATA VARIANCE	FILTERED DATA VARIANCE	PROBABLE MEAN DATA ERROR
9	2265.29	129	26.62	5.46	.0032
10	1330.42	128	23.05	5.12	.0030
11	894.85	128	25.23	5.31	.0033
12	695.73	128	22.68	5.11	.0029
13	972.96	127	23.70	4.88	.0029
14	958.98	126	22.03	4.705	.0026
15	1019.04	126	22.63	3.14	.0030
16	1304.51	126	22.12	3.11	.0029
17	3983.87	125	21.15	3.13	.0025
18	7537.51	125	21.68	5.05	.0029
19	-	125	20.68	5.18	.0027

6.2 MAXIMUM PEAK COMPENSATION

Power spectral estimates were generated for range gates 2 through 25 by the sequential application of the unbiased weighted mean of minimum variance estimator and normal probability filter. The question of maximum peak compensation must be considered. This requirement is mandated for two distinct reasons. The first being that maximum power spectral peaks are not generally symmetric about their maximum values. Secondly, the normal probability filter imposes the constraint that isolated raw data peaks are a low probability event. Consequently, these erroneous peaks are removed, which tends to aggravate the degree of symmetric peak uniformity.

To compensate for this lack of maximum peak symmetry, Green's Theorem can be efficiently used to compute the peak area and center of gravity by forming a closed region around the peak from the localized piece-wise values. For example, consider the illustrative peak configuration of Figure 6.2-1.

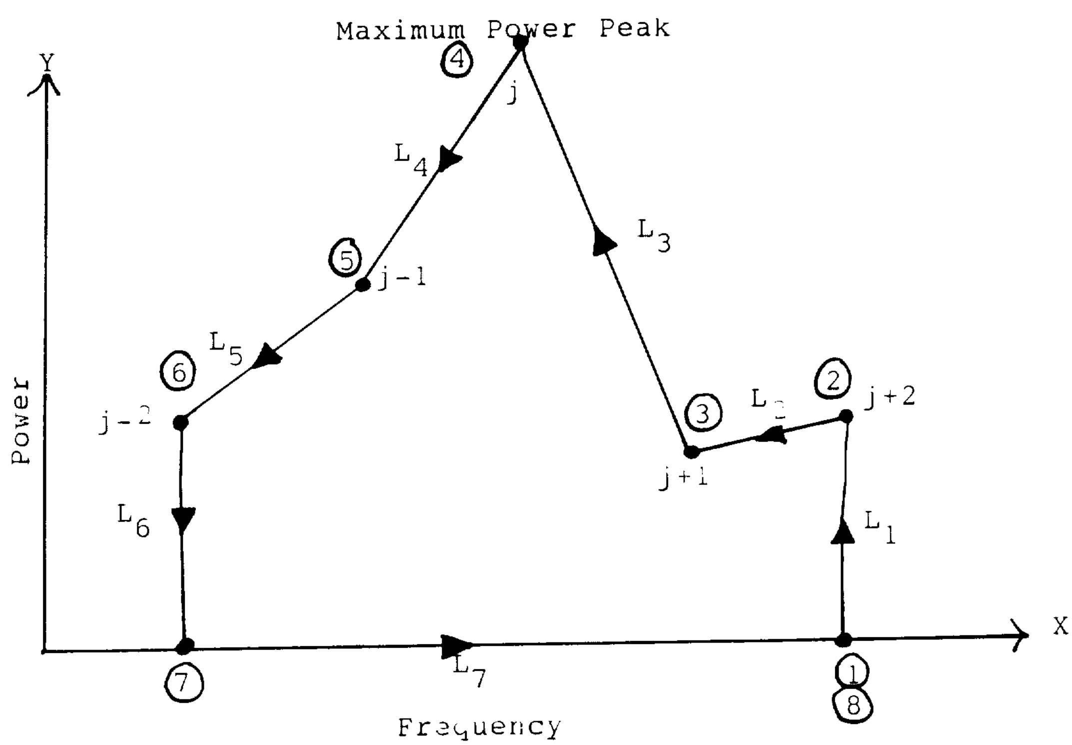
To compute the area of this closed region from Green's Theorem,

$$\int_C [P(x,y)dx + Q(x,y)dy] = \iint_R \left[\frac{dP(x,y)}{dx} - \frac{dQ(x,y)}{dy} \right] dxdy.$$

Where, R is the piece-wise closed region formed by traversing the indicated points in the counter-clockwise direction and C is the simple closed curve of the region boundary formed by the connecting straight lines,

$$L_i \equiv y = m_i x + b_i \quad \text{for } i = 1,2,\ldots 7.$$

FIGURE 6.2-1 CLOSED REGION AROUND MAXIMUM POWER PEAK



Here, $P(x,y) = 0$ and $Q(x,y) = x$ so that

$$\int_C m_i x\,dx = \iint_R dx\,dy = \text{Area}.$$

For the i^{th} line segment,

$$A_i = \frac{m_i x^2}{2} \Bigg|_{x_i}^{x_{i1}} = \frac{1}{2} \left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i} \right)\left(x_{i+1}^2 - x_i^2 \right).$$

Upon summing these segments over i, the total area, A, of the closed region becomes:

$$A = \frac{1}{2} \sum_{i=1}^{7} (y_{i+1}-y_i)(x_{i+1}+x_i).$$

It is interesting to note that the center of gravity, Æ , of this area can be determined from Green's Theorem by setting $P(x,y) = 0$ and $Q(x,y) = x^2$.

Therefore,

$$\int_C m_i x^2 dx = \iint_R 2x\,dx\,dy = Æ_i$$

so that $Æ_i$ has the form

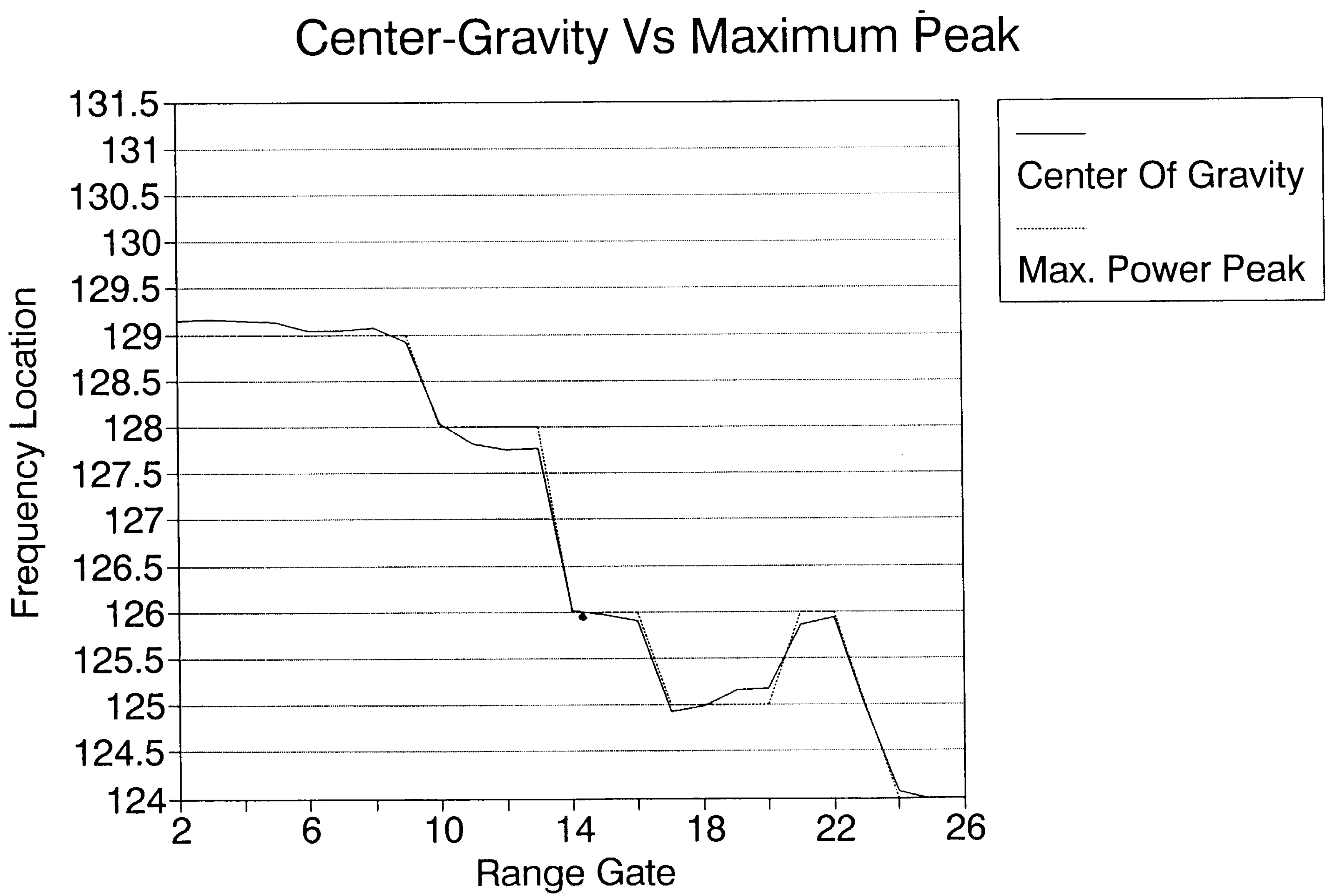
$$Æ_i = \frac{m_i x^3}{6} \Bigg|_{x_i}^{x_{i+1}} = \frac{1}{6} \left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i} \right)\left(x_{i+1}^3 - x_i^3 \right)$$

or

$$Æ = \frac{1}{6A} \sum_{i=1}^{7} (y_{i+1}-y_i)(x_{i+1}^2+x_i x_{i+1}+x_i^2).$$

Here, Æ , is the frequency location of the Doppler Shift associated with the maximum power spectral peak of the selected range gate. Figure 6-2-2 contains a graph of the center of gravity compensation relative to the maximum range gate power peaks for beam 1 associated with range gates 2 through 25.

FIGURE 6.2-2 BEAM 1: MAXIMUM PEAK CENTER OF GRAVITY COMPENSATION



To illustrate the comparative relationship between these computed values and measured balloon velocities, the computed and measured values are expressed in terms of their distance from the central frequency. Figure 6-2-3 provides a graphic of their central frequency comparison. When viewing this comparison, it must be remembered that the measured balloon velocities have both a time interval and an inertia bias.

6.3 LINEAR CORRELATION ASSUMPTION OF ADJACENT RANGE GATE

To assess the statistical assumption of linearly correlated wind velocity with respect to adjacent range gates, a series of beam 3 range gate results were analyzed. Here, beam 3 was selected because these power spectral estimates have a much larger velocity resolution capability, i.e., 6.7m/s.

The summarized results of this analysis can be illustrated by considering range gate 70 with three surrounding range gates on either side of range gate 70. Figure 6.3-1 denotes the raw vs. filtered range gate power for this example. Here, the isolated raw central data peak is located at frequency bin 130, with a power amplitude of 5.09. However, this peak is washed out (cancelled) by the lack of non-linear correlations in the spectral power amplitudes of the adjacent range gates shown in Figure 6.3-2. These non-linear spectral power amplitude correlations are presented in Figures 6.3-3, 6.3-4, and 6.3-5 for three, two and one surrounding range gates; respectively. Further amplification is provided by the computation of the maximum power peak centroid values listed in Table 6.3-1.

TABLE 6.3-1
COMPARATIVE PEAK CENTROIDS AS A FUNCTION OF SURROUNDING RANGE GATES

RANGE GATE 70
BEAM 3

SURROUNDING RANGE GATES	COMPUTED MAXIMUM PEAK CENTROID
1	123.91858
2	124.50734
3	124.72048

7.0 CONCLUSIONS AND RECOMMENDATIONS

The specific objectives of this study were: 1) to recommend a preferred algorithm for improving atmospheric signals that are occasionally obscured by natural and man-made interference; and 2) to demonstrate that the preferred statistical Normal Probabil-

FIGURE 6.2-3 COMPARISON OF COMPUTED RADIAL WIND VELOCITIES TO MEASURED BALLOON VELOCITIES

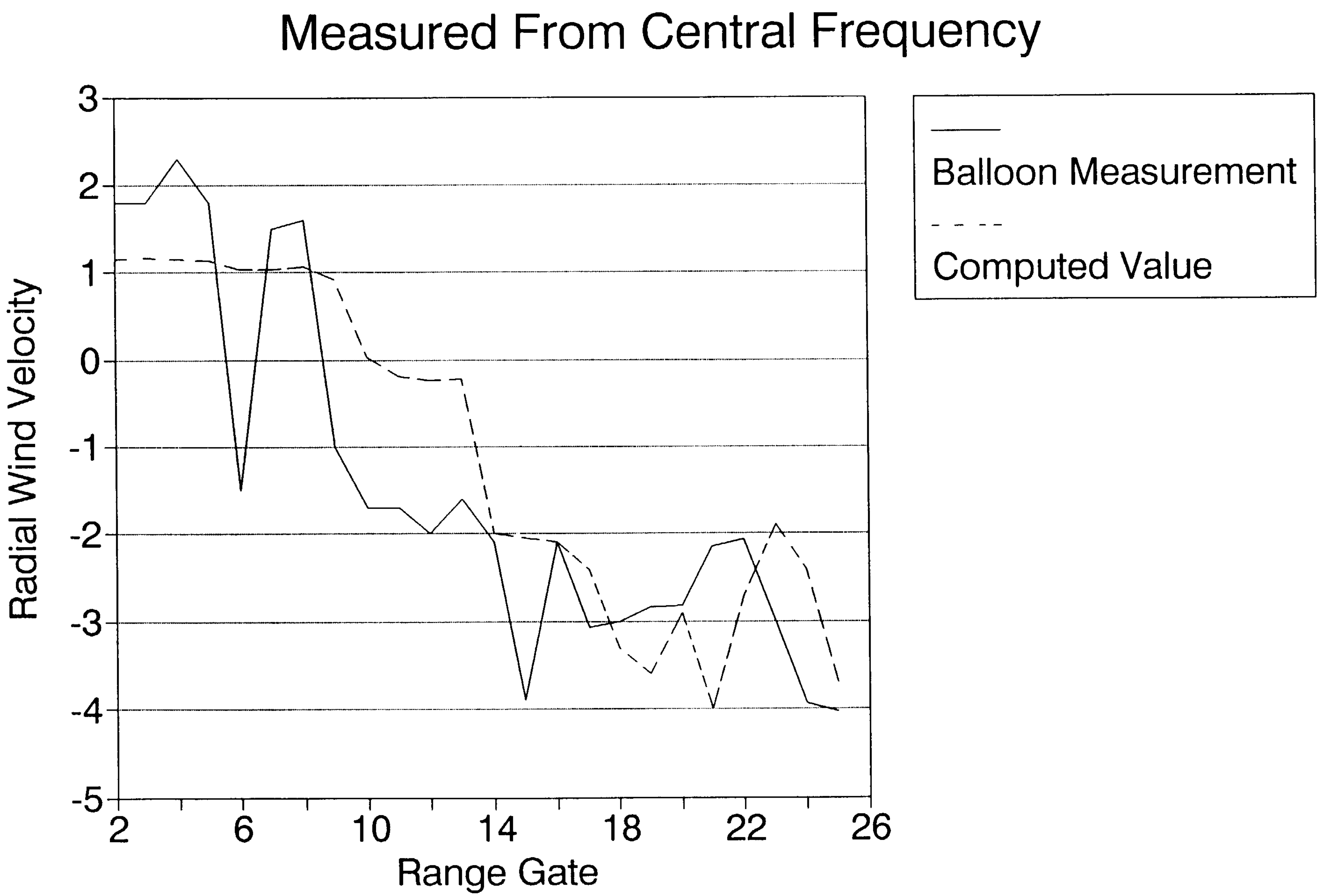


FIGURE 6.3-1 RAW vs FILTERED RANGE GATE POWER FOR BEAM 1: WITH 3 SURROUNDING RANGE GATES

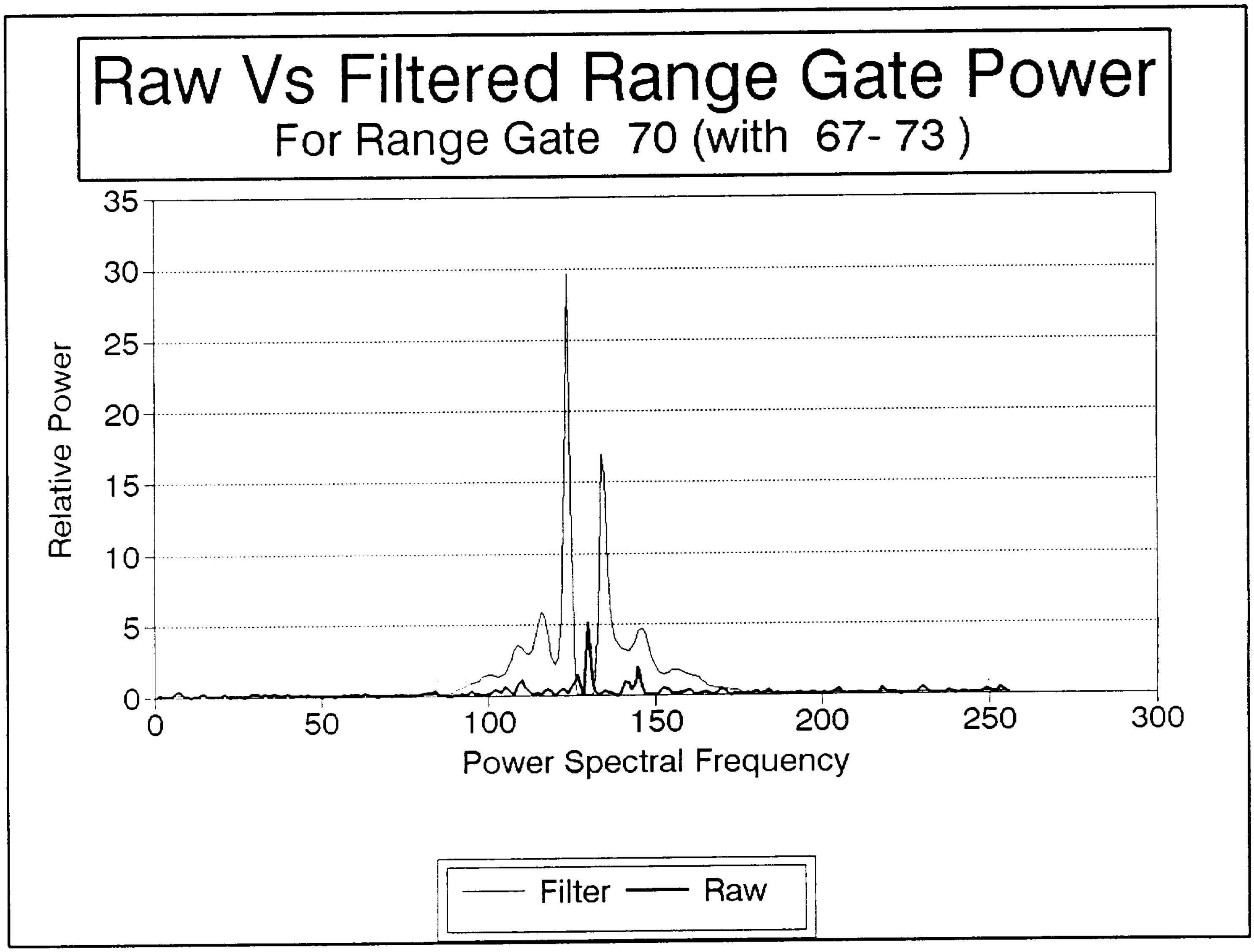


FIGURE 6.3-2 ENLARGEMENT OF FIGURE 6.3-1 DATA

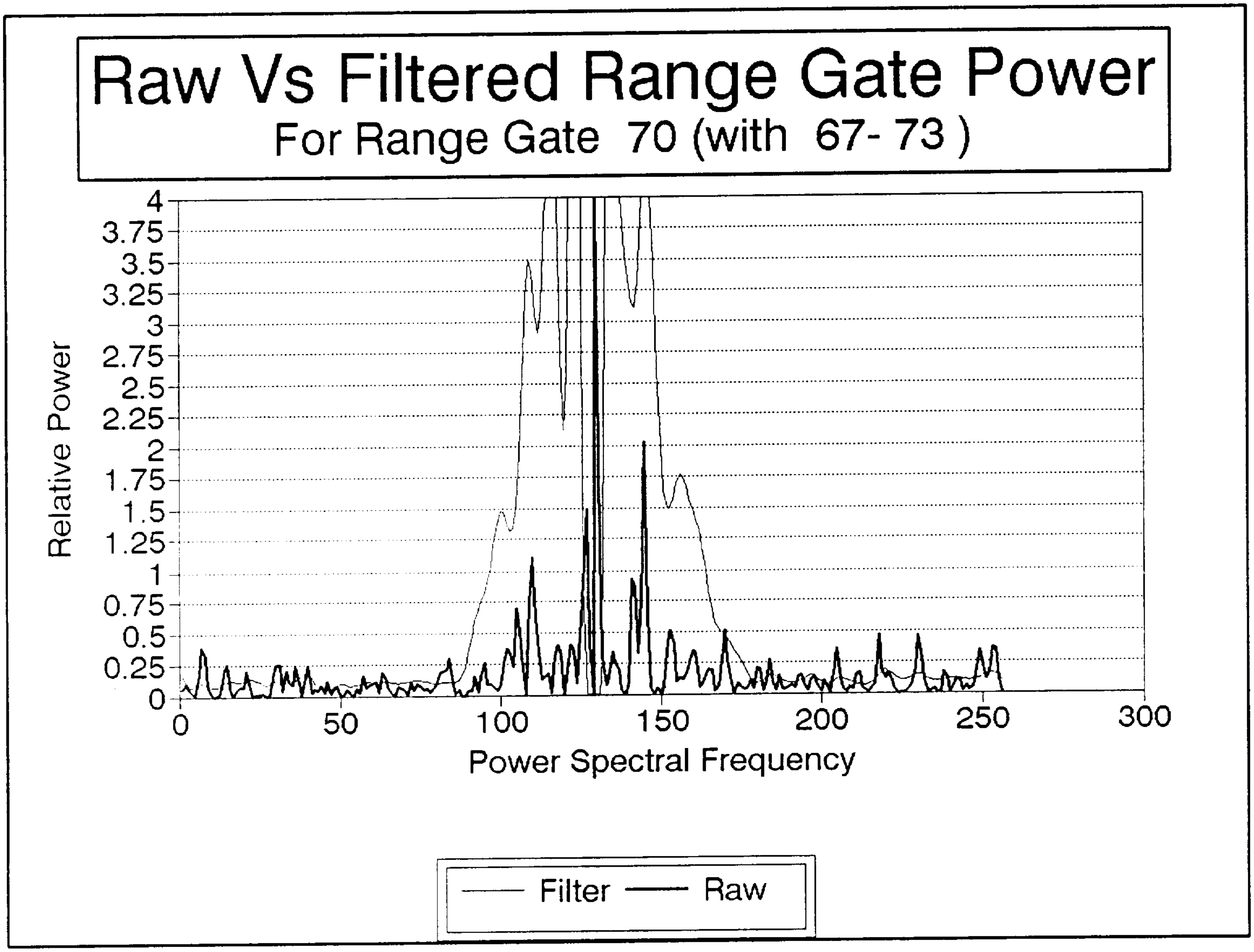


FIGURE 6.3-3

Raw Range Gate Data For:

Range Gates 67, 70, And 73

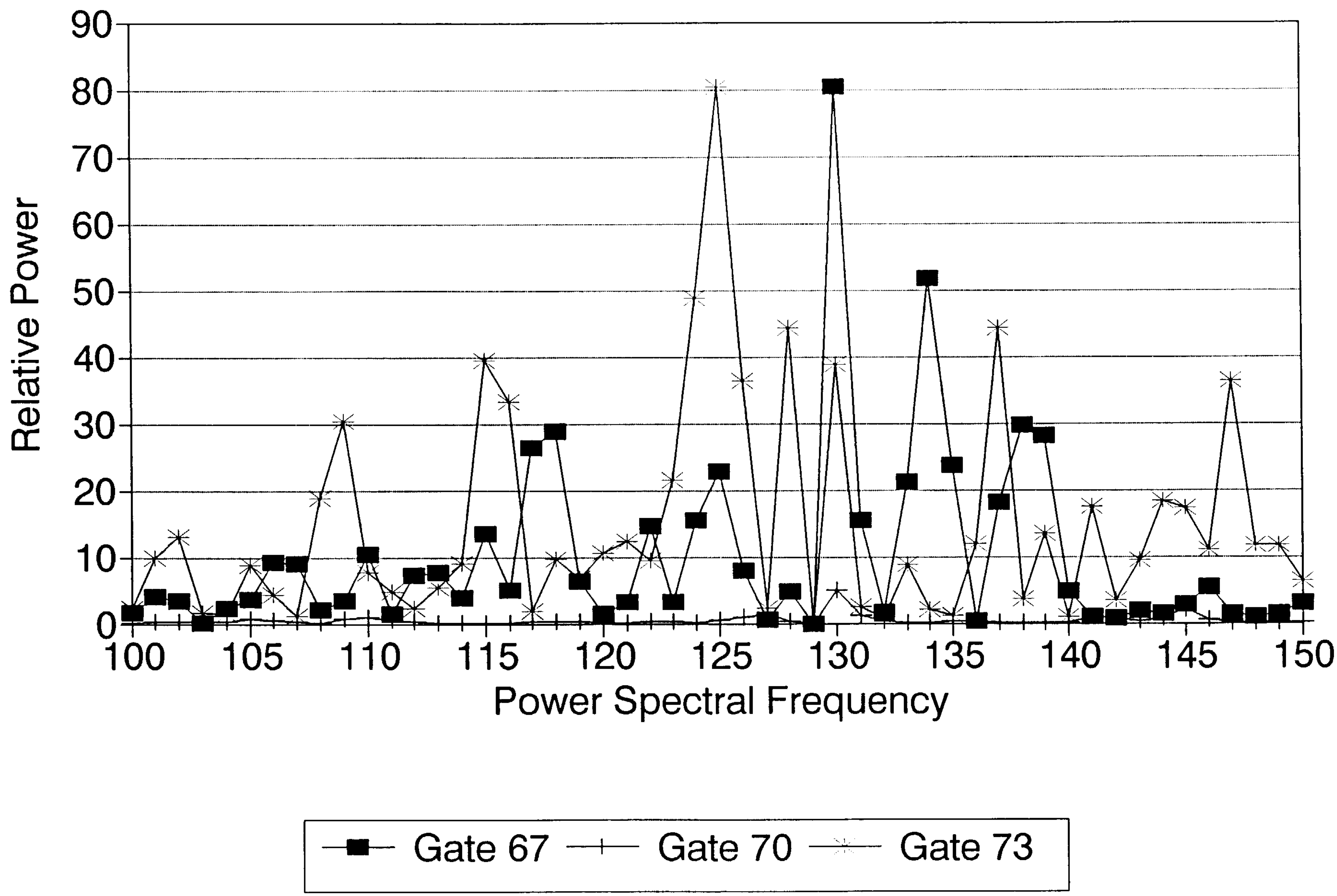


FIGURE 6.3-4

Raw Range Gate Data For:

Range Gates 68, 70, And 72

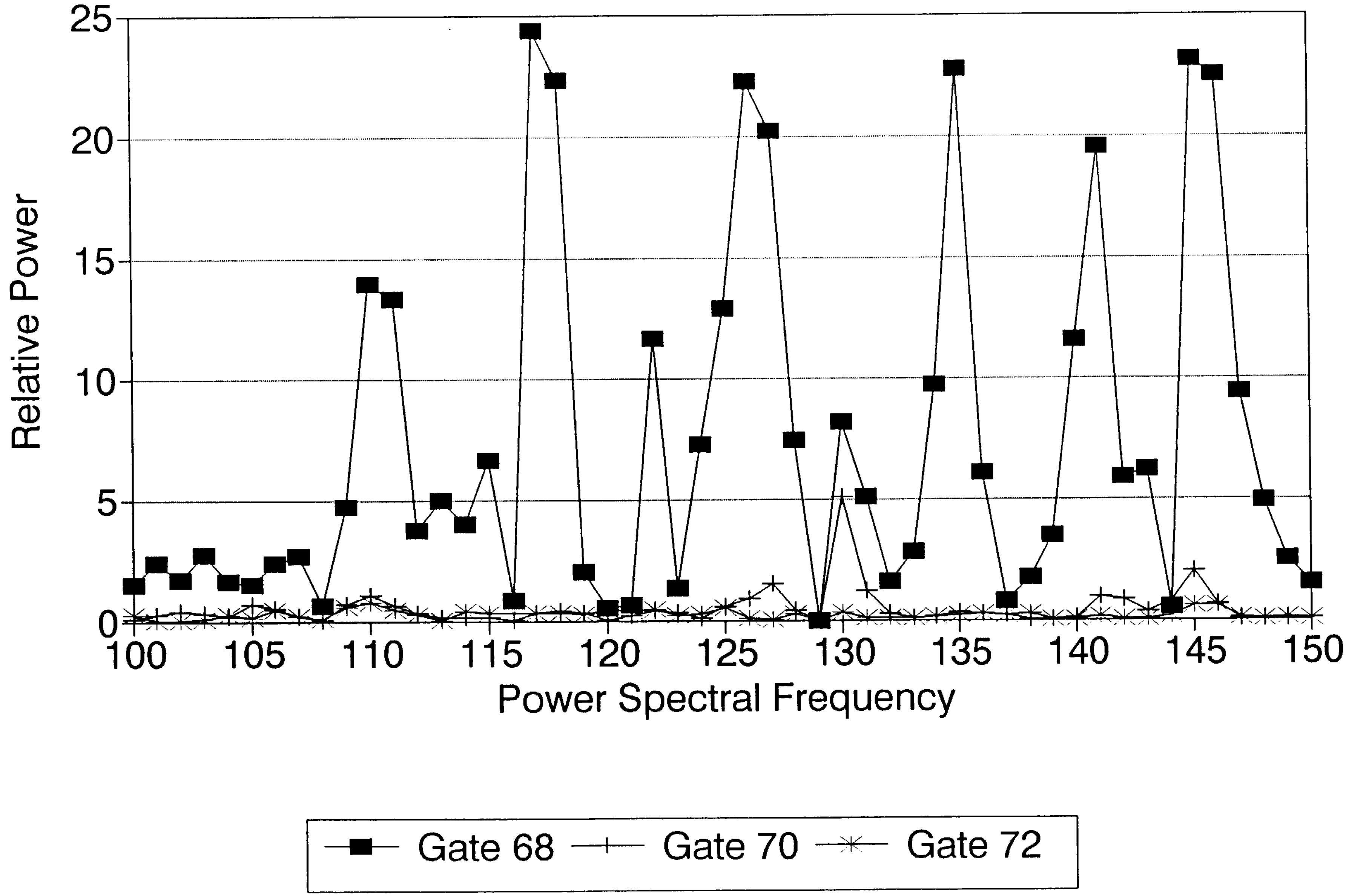
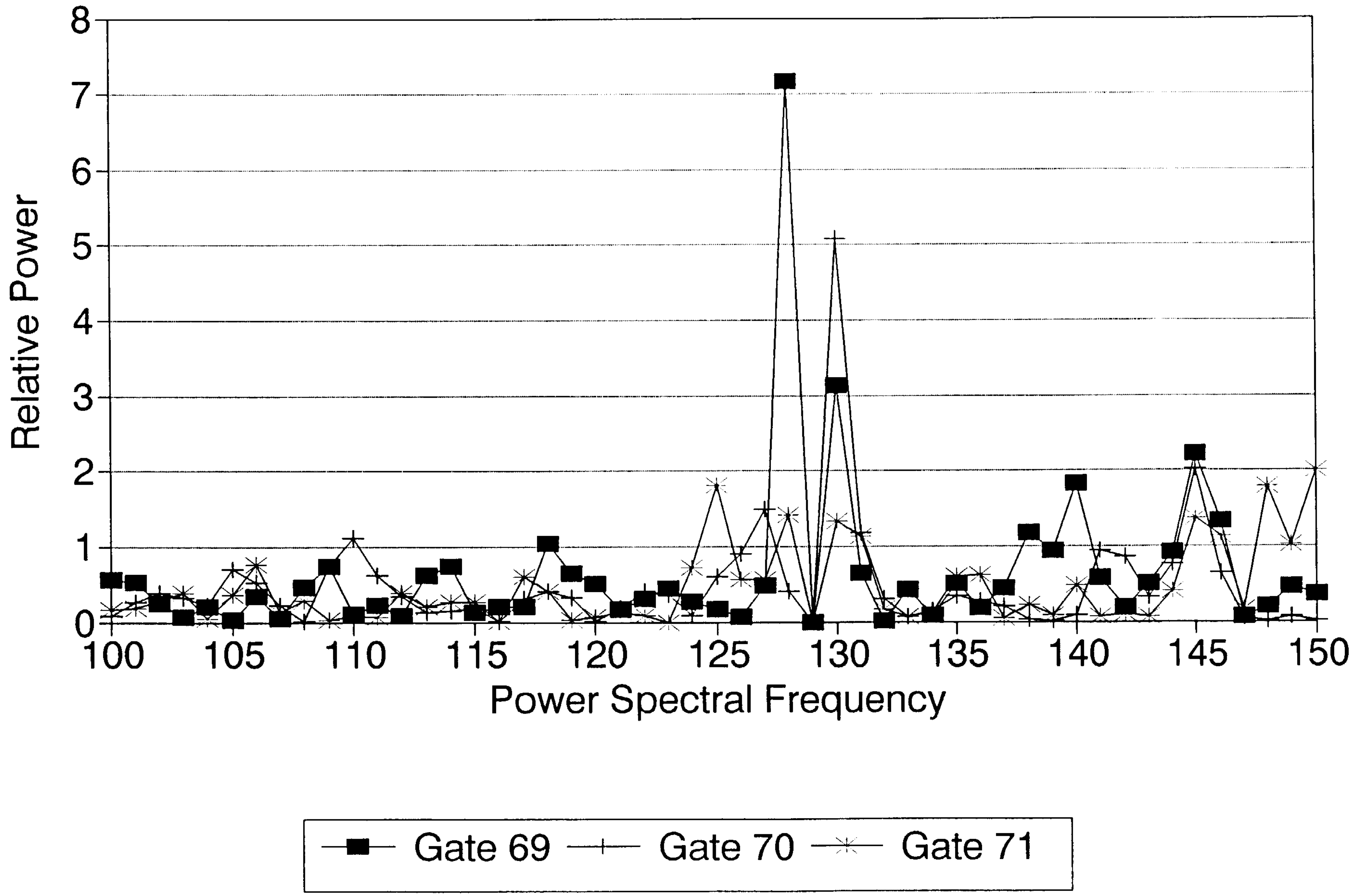


FIGURE 6.3-5 Raw Range Gate Data For:

Range Gates 69, 70, And 71



ity Filtering (SNPF) algorithm used in conjunction with the weighted mean of minimum variance estimator provides improved reliability of wind velocities.

Based on the analytical data results obtained from a prototype computer program for beam 1 and range gates 2 through 25 of the archived data file 14130612.90P, the computed radical wind velocities appear to be in correspondence with similar velocities obtained from balloon measurements. See Figure 6-2-3 for a comparison of these results. When viewing this comparison, one should realize that the measured balloon velocities suffer from inertia bias and wind dynamic effects.

The recommendations for further study are:

(1) The present SNPF algorithm has been tested against one limited set of wind predictions. Clearly, an extensive comparison of the SNPF predictions with all range gates and for numerous noise and interference conditions is required.

(2) In addition, a study is required that includes the identification of capabilities and limitations of the basic wind profiler and balloon measurements, such as balloon dynamics in wind. This information will allow an understanding of the intricacies of both wind measurement methods and potential explanations of the differences in the predictions.

APPENDIX A

PROTOTYPE FORTRAN SOURCE
CODE

(Statistical Normal Probability Filtering (SNPF) Programs)

```
      program neps_filter
      implicit real*8 (a-h,o-z)
      dimension jmode(3),araw(258,11),var(11),a(11),xbar(256)
      character*28 str,st
      character*13 file1,file2,file3
      character*2 cr
      character*1 yn,ny
      data jmode/0,4816,9632/
      cr = 'm'
      vtol = 1.5d-10
      iup = 112
      ny = 'y'
 5    print*,'************************************'
      print*,'** Enter Name Of Input Data File **'
      print*,'************************************'
      print*,' '
      read(*,900) file2
      l = index(file2,' ')-1
      file1 = file2(:l)
      if(i.gt.12) then
        print*,' '
        print*,'Named  Input  Data  File  Must  Have  12 Or Less'
        print*,'Alphanumeric Characters Including The Extension.'
        print*,'Do  You  Wish  To Enter Another File Name (y,n)?'
        read(*,901) yn
        if(yn.eq.ny) goto 5
        goto 999
      endif
      print*,'Requested Input Data File Name Is: ',file1
      open(8,file=file1,access='direct',status='old',form='formatted',
     1err = 35,iostat=iflag,recl=79)
 10   print*,' '
      print*,'Select Beam Mode (1,2,3)?'
      read*,mode
      print*,'Selected Beam Mode Equals: ',mode
      if((mode.ge.1).and.(mode.le.3)) then
 15     print*,' '
        print*,'Enter Desired Range Gate (1-112).'
        read*,j
        irange = 2000+(j-1)*150
        print*,'Requested Range Gate Equals: ',j
        print*,'At Height: ',irange,' (Meters)'
        if((j.lt.1).or.(j.gt.112)) then
          print*,'Entered Range Gate Out of Bound.'
          print*,'Must Be In The Closed Interval [1,112]'
          print*,'Do You Wish To Renter This Number (y,n)?'
          read*,yn
          if(yn.eq.ny) goto 15
          goto 999
        endif
 20     print*,' '
        print*,'Enter  Number  Of Surrounding Range Gates (1-5)'
        print*,'To Be Considered In The Statistical Data Filter'
        print*,'Smoothing Process.'
        read*,i
        print*,'Selected Number Of Surrounding Range Gates Equals: ',i
        if((i.lt.1).or.(i.gt.5)) then
          print*,'Selected Number Of Range Gates Out Of Bound.'
          print*,'Must Be In The Closed Interval [1,5]'
 25       print*,'Do You Wish To Reenter This Number (y,n)?'
          read*,yn
          if(yn.eq.ny) goto 20
          goto 999
        endif
      else
        print*,'Selected Beam Mode Is Out Of Bound.'
```

```
          print*,'Must Be In The Closed Interval [1,3]'
          print*,'Do You Wish TO Reenter This Number (y,n)?'
          read*,yn
          if(yn.eq.ny) goto 10
          goto 999
        endif
        il = j-i
        ih = j+i
        iouth = i
        if(ih.gt.iup) iouth = iup-j
        ioutl = i
        if(il.lt.1) ioutl = j-1
        k = min0(ioutl,iouth)
        if(k.lt.i) then
          print*,'Requested  Number  Of Surrounding Range'
          print*,'Gates  Cannot  Be  Supported  Using The'
          print*,'Currently  Specified  Range Gate Value.'
          print*,'This  Number Must Be Less Than Or Equal'
          print*,'To ',k
          goto 25
        endif
        print*,' '
        print*,'Screen Print Of Incrementally Processed Data (y,n)?'
        read*,yn
        iwrt = 0
        if(yn.eq.ny) iwrt = 1
        igate = 1
        kk = 256
        do istart = il,ih
          print*,' '
          if(iwrt.eq.1) print*,'Raw Data For Range Gate ',istart
          icount = 1
          iend = 43*istart+jmode(mode)
          ibegin = iend-42
          do l = ibegin,iend
            read(8,902,rec=l,iostat=iflag,err=30) (a(i),i=1,6)
            do i = 1,6
              araw(icount,igate) = a(i)
              if((iwrt.eq.1).and.(icount.le.kk)) then
                print*,'      ',icount,'  ',a(i)
              endif
              icount = icount+1
            end do
          end do
          igate = igate+1
        end do
        igate = igate/2
c       utilize NEPS to obtain raw power spectral range  gate
c       data error variances
        k = 1
        kk = 256
        do i = il,ih
          do j = 1,kk
c           temporarily load the raw power spectral range gate
c           data into the xbar array.
            xbar(j) = araw(j,k)
          end do
          call neps(xbar,kk,segma,pme,ierr)
          var(k) = segma*segma
          print*,'Data Variance/Probable Error For Range Gate ',i
          print*,'      Variance = ',var(k)
          print*,'      Probable Error = ',pme
          k = k+1
        end do
c       compute unbiased weighted mean of minimum variance weighting
c       coefficients and variance estimates.
```

```
      ngates = ih-il+1
      sum = 0.0d0
      do j = 1,ngates
        if(var(j).le.vtol) var(j) = vtol
        sum = sum+(1.0d0/var(j))
      end do
      if(iwrt.eq.1) then
        print*,' '
        print*,'Unbiased Range Gate Coefficient Weights:'
      endif
      ip = il
      vxbar =0.0d0
      do j = 1,ngates
        a(j) = 1.0d0/(var(j)*sum)
        vxbar = vxbar+a(j)*a(j)*var(j)
        if(iwrt.eq.1) then
          print*,ip,'th. Range Gate Coefficient = ',a(j)
          ip = ip+1
        endif
      end do
c     compute weighted mean of minimum variance estimates
      do j = 1,kk
        xbar(j) = 0.0d0
        do i = 1,ngates
          xbar(j) = xbar(j)+a(i)*araw(j,i)
        end do
      end do
      i = (il+ih)/2
      print*,' '
      print*,'Weighted Mean Of Minimum Variance Equals: ',vxbar
      print*,'Iterated Data Error Variance: (range gate ',i,' )'
c     statistically  filter the weighted mean of minimum variance
c     estimates down to a data error variance of vxbar
      call neps(xbar,kk,segma,pme,ierr)
      vfmmv = segma*segma
      print*,'      ',vfmmv
      do while (vfmmv.gt.vxbar)
        call neps(xbar,kk,segma,pme,ierr)
        vfmmv = segma*segma
        print*,'      ',vfmmv
      end do
      print*,' '
c     compute frequency index at maximum power
      xmax = xbar(1)
      jmax = 1
      do j = 2,kk
        if(xbar(j).gt.xmax) then
          xmax = xbar(j)
          jmax = j
        endif
      end do
      dopsmp = 128-jmax
      print*,'Doppler Shift At Maximum Power Peak = ',dopsmp
      print*,'Power At Maximum Peak = ',xmax
      print*,' '
c     compute center of gravity of maximum power peak using
c     green's theorem.
      ip = jmax+2
      a(1) = dble(ip)
      var(1) = 0.0d0
      do j = 2,6
        a(j) = dble(ip)
        var(j) = xbar(ip)
        ip = ip-1
      end do
      a(7) = a(6)
```

```
      var(7) = 0.0d0
      area = 0.0d0
      vel = area
      do j = 1,6
        j2 = j+1
        temp = var(j2)-var(j)
        area = area+temp*(a(j2)+a(j))
        vel = vel+temp*(a(j2)*a(j2)+a(j)*a(j2)+a(j)*a(j))
      end do
      area = 0.5d0*area
      vel = (1.0d0/(6.0d0*area))*vel
      print*,'Radial Wind Velocity = ',vel
      print*,' '
      if(iwrt.eq.1) then
        print*,'Filtered Mean Of Minimum Variance Estimators'
        do j = 1,kk
          print*,'   ',j,'  ',xbar(j)
        end do
      endif
c     write ASCII comma and "" delimited file for QUATTO PRO plotting.
      l = index(file1,'.')
      file2 = file1(:l)//'plt'
      file3 = file1(:l)//'tem'
      print*,' '
      print*,'Currently Generating File For'
      print*,'Later QUATTO PRO Plotting.'
      print*,'Plot File Name Equals: ',file2
      open(9,file=file3,access='direct',status='scratch',
     1form='formatted',recl=26)
      open(10,file=file2,access='sequential',status='unknown',
     1form='unformatted')
c     write required QUATTO PRO plotting series,
c     axes and text legends
      i = (il+ih)/2
      st(1:2) = cr
      write(10,906) st(1:2)
      write(10,907) i,il,ih,st(1:2)
      write(10,908) st(1:2)
      write(10,909) st(1:2)
      write(10,910) st(1:2)
      write(10,911) st(1:2)
      do j = 1,kk
        write(9,903,rec=j) j,xbar(j),araw(j,igate)
      end do
      do j = 1,kk
        read(9,904,rec=j) st(:26)
        str(:28) = st(:26)//cr
        write(10,905) str(:28)
      end do
      close(unit=10)
      close(unit=8)
      goto 999
 30   print*,' '
      if(iflag.eq.-1) then
        print*,' '
        print*,'End-Of-File Condition Has Ocurred'
        print*,'On Input Data File: ',file1
        goto 999
      else
        print*,'Read Operation on Input Data File:'
        print*,'             (',file1,')'
        print*,'Has Been Terminated With A Standard'
        print*,'FORTRAN Run-time Error Of: ',iflag
        goto 999
      endif
 35   print*,'Unknown Problem With Requested Input Data File.'
```

```
      print*,'      (',file1,')'
      print*,'Must Terminate File Processing.'
900   format (a13)
901   format (a1)
902   format (6(e13.6),1x)
903   format (i8,2(',',f8.2))
904   format (a26)
905   format (a28)
906   format (34h"Raw Vs Filtered Range Gate Power",a2)
907   format (16h"For Range Gate ,i3,7h (with ,i3,1h-,i3,3h )",a2)
908   format (26h"Power Spectral Frequency",a2)
909   format (16h"Relative Power",a2)
910   format (8h"Filter",a2)
911   format (5h"Raw",a2)
999   stop
      end
```

```
      SUBROUTINE NEPS(Y,N,SEGMA,PME,IER)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1072),Y(1000),BU(39),AJ(173),JA(18),AK(153),KA(12)
      PARAMETER (FACT = 0.6744385D0)
C
C     NEPS LAST MODIFIED 12-17-91
C
      DATA KA/1,35,36,66,67,93,94,117,118,137,138,153/
C    ##############################################################12
      DATA JA/100,1,39,50,40,74,20,75,104,10,105,131,4,132,154,1,155,
     $ 173/
C    ##############################################################18
      DATA AK/0.1570D0,0.1482D0,0.1254D0,0.0948D0,0.0628D0,0.0341D0,
     $ 0.0117D0,-0.0035D0,-0.0119D0,-0.0148D0,-0.0138D0,-0.0108D0,
     $ -0.0070D0,-0.0034D0,-0.0005D0,0.0014D0,0.0023D0,0.0025D0,
     $ 0.0022D0,0.0017D0,0.0010D0,0.0005D0,0.0000D0,-0.0002D0,
     $ -0.0004D0,-0.0004D0,-0.0003D0,-0.0002D0,-0.0001D0,-0.0001D0,
     $ 0.0000D0,0.0000D0,0.0001D0,0.0001D0,0.0001D0,
C    ########################################35###################
     $ 0.1769D0,0.1644D0,0.1329D0,0.0928D0,0.0536D0,0.0216D0,
     $ -0.0004D0,-0.0125D0,-0.0165D0,-0.0151D0,-0.0110D0,-0.0062D0,
     $ -0.0021D0,0.0008D0,0.0024D0,0.0028D0,0.0025D0,0.0018D0,
     $ 0.0010D0,0.003D0,-0.0001D0,-0.0004D0,-0.0004D0,-0.0004D0,
     $ -0.0003D0,-0.0002D0,0.0000D0,0.0000D0,0.0001D0,0.0001D0,
     $ 0.0001D0,
C    #####66#######################################################
     $ 0.2076D0,0.1873D0,0.1397D0,0.0842D0,0.0359D0,0.0027D0,
     $ -0.0145D0,-0.0191D0,-0.0161D0,-0.0099D0,-0.0038D0,0.0005D0,
     $ 0.0027D0,0.0032D0,0.0026D0,0.0016D0,0.0006D0,-0.0001D0,
     $ -0.0004D0,-0.0005D0,-0.0004D0,-0.0002D0,-0.0001D0,0.0000D0,
     $ 0.0001D0,0.0001D0,0.0001D0,
C    ######################93#######################################
     $ 0.2347D0,0.2056D0,0.1412D0,0.0721D0,0.0191D0,-0.0110D0,
     $ -0.0211D0,-0.0186D0,-0.0110D0,-0.0035D0,0.0014D0,0.0034D0,
     $ 0.0033D0,0.0022D0,0.0009D0,0.0000D0,-0.0004D0,-0.0005D0,
     $ -0.0004D0,-0.0002D0,0.0000D0,0.0000D0,0.0001D0,0.0001D0,
C    ####################################################117#######
     $ 0.22771D0,0.2297D0,0.1357D0,0.0486D0,-0.0046D0,-0.0236D0,
     $ 0.0211D0,-0.0108D0,-0.0017D0,0.0031D0,0.0039D0,0.0027D0,
     $ 0.0010D0,-0.0001D0,-0.0005D0,-0.0005D0,-0.0003D0,-0.0001D0,
     $ 0.0001D0,0.0001D0,
C    #############137##############################################
     $ 0.3601D0,0.2604D0,0.1045D0,0.0023D0,-0.0293D0,-0.0213D0,
     $ -0.0056D0,0.0032D0,0.0044D0,0.0023D0,0.0003D0,-0.0006D0,
     $ -0.0005D0,-0.0002D0,0.0000D0,0.0001D0/
C    ##################################153#######################173
      DATA AJ/0.9155D0,0.4491D0,0.1321D0,-0.0563D0,-0.1442D0,-0.1619D0,
     $ -0.1374D0,-0.0937D0,-0.0474D0,-0.0084D0,0.0185D0,0.0330D0,
     $ 0.0369D0,0.0334D0,0.0257D0,0.0167D0,0.0083D0,0.0017D0,-0.0027D0,
     $ -0.0049D0,-0.0055D0,-0.0049D0,-0.0037D0,-0.0023D0,-0.0010D0,
     $ -0.0001D0,0.0005D0,0.0008D0,0.0009D0,0.0007D0,0.0005D0,0.0003D0,
     $ 0.0001D0,0.0000D0,-0.0001D0,-0.0001D0,-0.0001D0,-0.0001D0,
     $ -0.0001D0,1.0237D0,0.4337D0,0.0626D0,-0.1293D0,-0.1917D0,
     $ -0.1745D0,-0.1199D0,-0.0579D0,-0.0065D0,0.0268D0,0.0418D0,
     $ 0.0425D0,0.0345D0,0.0229D0,0.0114D0,0.0024D0,-0.0034D0,-0.0060D0,
     $ -0.0063D0,-0.0051D0,-0.0034D0,-0.0017D0,-0.0003D0,0.0006D0,
     $ 0.0010D0,0.0010D0,0.0008D0,0.0005D0,0.0003D0,0.0000D0,-0.0001D0,
     $ -0.0002D0,-0.0002D0,-0.0001D0,-0.0001D0,1.1847D0,0.3812D0,
     $ -0.0600D0,-0.2297D0,-0.2309D0,-0.1542D0,-0.0633D0,0.0069D0,
     $ 0.0452D0,0.0548D0,0.0460D0,0.0293D0,0.0127D0,0.0005D0,-0.0060D0,
     $ -0.0079D0,-0.0067D0,-0.0043D0,-0.0018D0,0.0000D0,0.0010D0,
     $ 0.0012D0,0.0010D0,0.0007D0,0.0003D0,0.0000D0,-0.0001D0,-0.0002D0,
     $ -0.0002D0,-0.0001D0,1.3209D0,0.3082D0,-0.1750D0,-0.2965D0,
     $ -0.2294D0,-0.1060D0,-0.0027D0,0.0531D0,0.0654D0,0.0512D0,
     $ 0.0281D0,0.0078D0,-0.0045D0,-0.0091D0,-0.0083D0,-0.0052D0,
     $ -0.0020D0,0.0003D0,0.0013D0,0.0014D0,0.0010D0,0.0005D0,0.0001D0,
```

```
     $ -0.0002D0,-0.0002D0,-0.0002D0,-0.0001D0,1.5204D0,0.1523D0,
     $ -0.3468D0,-0.3471D0,-0.1700D0,-0.0075D0,0.0731D0,0.0810D0,
     $ 0.0523D0,0.0189D0,-0.0031D0,-0.0112D0,-0.0100D0,-0.0053D0,
     $ -0.0011D0,0.0012D0,0.0017D0,0.0012D0,0.0005D0,0.0000D0,
     $ -0.0002D0,-0.0002D0,-0.0001D0,1.8615D0,-0.2545D0,-0.5842D0,
     $ -0.2660D0,0.0302D0,0.1257D0,0.0899D0,0.0276D0,-0.0087D0,
     $ -0.0156D0,-0.0088D0,-0.0013D0,0.0020D0,0.0019D0,0.0008D0,
     $ 0.0000D0,-0.0003D0,-0.0002D0,-0.0001D0/
C
C     DETERMINE MINIMUM DATA OBSERVATION VALUE SO THAT
C     THE DATA CAN BE TRANSFORMED,(I.E.,Y'(I) = Ln(Y(I)-YMIN),
C     SO THAT ITS MEASURE OF PRECISION HAS NEARLY THE SAME
C     VALUE FOR ALL OBSERVATION VALUES.
C
      YMIN = Y(1)
      DO I = 2,N
        YMIN = 0.5D0*(YMIN+Y(I)-DABS(YMIN-Y(I)))
      END DO
      YMIN = YMIN-2.0D-08
C
C     CHECK ADEQUACY OF OBSERVATION DATA SAMPLE.
C
      IF(N.GT.4) THEN
        SUM = 0.0D0
        DO I = 2,N-2
          R = (Y(I)-YMIN)/(Y(I+1)-YMIN)
          S = (Y(I+2)-YMIN)/(Y(I-1)-YMIN)
          A(I+2) = DLOG(S)+3.0D0*DLOG(R)
          SUM = SUM+A(I+2)
        END DO
C
C       COMPUTE THIRD DIFFERENCE MEAN.
C
        SUM = SUM/DBLE(N-3)
C
C       COMPUTE TRANSFORMED PROBABLE THIRD DIFFERENCE ERROR
C       FROM THE NORMAL PROBABILITY INDEX OF PRECISION.
C
        PE = 0.0D0
        DO I = 4,N
          R = A(I)-SUM
          PE = PE+R*R
        END DO
        C1 = DBLE(N-3)
        C2 = DSQRT(PE)*FACT
        PE = C2/DSQRT(C1-1.0D0)
C
C       COMPUTE MEAN PROBABLE SMOOTHING ERROR REDUCTION.
C
        PME = C2*PE/(DSQRT(C1*C1-C1)*C1)
C
C       COMPUTE PROBABLE ERROR STANDARD DEVIATION.
C
        SEGMA = PE/FACT
      ELSE
C
C       INSUFFICIENT DATA SAMPLE FOR STATISTICAL DATA SMOOTHING.
C       MUST HAVE GREATER THAN FOUR, (4), DATA OBSERVATIONS.
C
        IER = 1
        RETURN
      ENDIF
      IER = 0
C
C     USING THE FUNDAMENTAL THEOREM OF INDUCTIVE PROBABILITY,
C     MAP THE OBSERVED THIRD DIFFERENCE PROBABLE DATA ERRORS
```

```
C      INTO THE SET OF SMOOTHING COEFFICIENTS.
C
       DO 65 I = 1,16,3
         EPS = 1.0D0/DBLE(JA(I))
         IF((PE.LE.EPS).OR.(I.EQ.16)) THEN
C
C          EXTRAPOLATE UPPER AUXILIARY POINT DATA.
C
           L = I+1
           M = JA(L+1)-JA(L)+1
           IF(M.GT.N) THEN
C
C            STATISTICAL DATA SAMPLE NOT LARGE ENOUGH TO PROVIDE
C            HIGH CONFIDENCE DATA SMOOTHING OR AUXILIARY DATA POINT
C            GENERATION. NOTICE, THE MINIMUM SAMPLE SIZE IS RETURNED
C            IN IER.
C
             IER = M
             M = N
           ENDIF
C
C          DETERMINE NUMBER OF REQUIRED AUXILIARY DATA POINTS.
C
           IP = (2*I+1)/3
           NK = KA(IP+1)-KA(IP)
           DO 10 K = 1,M
             BU(K) = DLOG(Y(K)-YMIN)
   10      CONTINUE
C
C          GENERATE AUXILIARY DATA POINTS WHICH SATISFY THE A PRIORI
C          PROBABILITY HYPOTHESIS THAT THE SUPPLIED DATA OBSERVATIONS
C          HAVE A NORMAL ERROR LAW DISTRIBUTION.
C
           M1 = M-1
           DO 25 K = NK,1,-1
             L1 = JA(L)
             SUM = AJ(L1)*BU(1)
             L1 = L1-1
             DO 15 J = 2,M
               SUM = SUM+AJ(L1+J)*BU(J)
   15        CONTINUE
             A(K) = SUM
C
C            REASSIGN UPPER AUXILIARY POINTS.
C
             DO 20 L1 = M1,1,-1
               BU(L1+1) = BU(L1)
   20        CONTINUE
             BU(1) = SUM
   25      CONTINUE
C
C          EXTRAPOLATE LOWER AUXILIARY POINT DATA.
C
           L1 = 1
           DO 30 K = N,N-M+1,-1
             BU(L1) = DLOG(Y(K)-YMIN)
             L1 = L1+1
   30      CONTINUE
           NK1 = NK+1
           DO 45 K = N+NK1,2*NK+N
             L1 = JA(L)
             SUM = AJ(L1)*BU(1)
             L1 = L1-1
             DO 35 J = 2,M
               SUM = SUM+AJ(L1+J)*BU(J)
   35        CONTINUE
```

```
            A(K) = SUM
C
C           REASSIGN LOWER AUXILIARY POINTS.
C
            DO 40 L1 = M1,1,-1
              BU(L1+1) = BU(L1)
   40       CONTINUE
            BU(1) = SUM
   45     CONTINUE
C
C         LOAD OBSERVATION DATA INTO THE A(J) ARRAY FOR
C         J = NK+1,NK+2,...,NK+N.
C
          DO 50 K = 1,N
            A(NK+K) = DLOG(Y(K)-YMIN)
   50     CONTINUE
C
C         IMPOSE CONSERVATION THEOREM SMOOTHING CONDITIONS:
C
C            1.) THE SUM OF RAW DATA OBSERVATIONS EQUALS THE
C                SUM OF SMOOTHED DATA OBSERVATIONS.
C
C            2.) THE RAW DATA MOMENTS OF ORDERS ONE, TWO, AND
C                THREE EQUAL THE MOMENTS OF THE SMOOTHED DATA.
C
C            3.) THE SMOOTHED AND UN-SMOOTHED DATA OBSERVATION
C                GRAPHS HAVE THE FOLLOWING CHARACTERISTICS:
C
C                  a) Equal areas,
C
C                  b) Same center of gravity, and
C
C                  c) Identical moments of inertia
C                     about any line parallel to the
C                     observation ordinates,e.g., the
C                     Y-axis.
C
          L1 = KA(IP)
          DO 60 J = NK1,NK+N
            SUM = AK(L1)*A(J)
            DO 55 K = 1,NK
              SUM = SUM+AK(L1+K)*(A(J+K)+A(J-K))
   55       CONTINUE
            Y(J-NK) = DEXP(SUM)+YMIN
c             Y(J-NK) = SUM
   60     CONTINUE
          RETURN
        ENDIF
   65 CONTINUE
      END
```